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10 February 1953

Mr W. K. Wallace, Member, in the Chair

The following Paper was presented for discussion and, on the motion of the Chairman, the thanks of the Division were accorded to the Authors.

Structural Paper No. 34

"Corrugated Concrete Shell Roofs"

by

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SYNOPSIS

The enclosure of space was the original aim and remains still one of the principal aims of civil engineering science. At present the provision of adequate shelter is severely impeded by steel shortage.

If current and future demands are to be met, the arch must be rescued from the comparative obscurity into which it has been relegated by the preference given to beams and girders.

Engineers are frequently unkind in their treatment of concrete, impolitely disregarding its aversion to tensile stress. In consequence it is too often said that "concrete roofs always crack." This is not the case.

The experiences of a quarter of a century's work in the development of the catenary concrete arch, in corrugated form, are described: cost, its possibilities, and the future outlook are discussed. By way of example it is shown that corrugated true-arch shells of 700 feet span, 4 inches thick, are practical and economical and that the drain on world steel supplies by the builder can be eased by more sympathetic use of the one material that is still plentiful—concrete.

Part 1 deals generally with methods.

Part 2 (which was prepared by Mr Aston) deals more particularly with the theoretical aspect.

An Appendix describes briefly a design for a hangar of 310 feet span.

Part 1

Corrugated Concrete Shell Roofs

by Mr Waller

Introduction

The past 30 years have brought a sharply rising demand for unobstructed covered space. The need for increased clearances is emphasized. Mechanization on the farm has put the old stable and farm-shed out of business; road transport calls for new centralized depots where previously scattered buildings met requirements; mass buying and new methods of stacking raise new problems in storage; industry continuously expands in the old countries and overseas, while the demands by air transport for shelter in every conceivable kind of climate present structural problems for which no obvious solution has yet appeared. Each step forward in science, each rise in living standards calls for more covered space—and it was ever thus.

Parallel with this demand is the world shortage of the modern wonder material, steel, and of the versatile and sympathetic timber which, since the dawn of history, has been such a standby. The position may improve but the demands of the mechanical engineer are so persistent that the former generous supplies available for structural work can scarcely be expected to reappear. Failing the discovery of adequate substitutes to fill the gap, better use must be made of the materials that remain in good supply; their advantages and shortcomings must be more closely studied and structural forms modified in the light of those studies. The frowns of vested interests should not be permitted to interfere with such development and it should be remembered that even established practice must submit to gradual disestablishment at the bidding of progress.

The influence of structural steel upon methods has been remarkable. In comparatively few years it completely changed the form of major buildings. When the Author's father entered the profession structural steel was unknown. No sooner had the Author's turn come to make a start than supplies began to cause anxiety. To redress the balance the French, to whom the engineers of the world owe so much, devised reinforced

concrete and later still, as the need to save steel further increased, the same imaginative race introduced prestressed concrete. The Germans, too, made their contribution and, with their characteristic diligence, evolved analyses of the intricate stresses that occur when reinforced-concrete members in the form of semi- or part-cylinders, acting as girders, are used as a roof covering. This system, usually called the barrel-vault or " shell " construction, has somewhat the appearance of the arch but it is not by nature an arch and should not be confused with the form of construction now presented for discussion.

The Author feels strongly that while this problem of enclosing space must be attacked from many different angles—indeed from every conceivable angle—simplicity in methods of calculation and construction should be given high priority. Buildings are required in all latitudes and in many cases the technical knowledge of those who must design them and the manual skill of those who must erect them are strictly limited. The mathematical prowess of those who revel in the somewhat mystical approximations of Herr Zeiss is denied to many members of the profession. The same is to some extent true in the case of prestressed concrete. This comment is not aimed at discounting the value of the work of those who have championed these systems, but is rather a plea that the needs of the many who have no ready access to specialist designers, man-power, and plant shall not be overlooked. One would go further. Is it not almost universally true that our more notable accomplishments, as they gradually progress towards perfection, are characterized by ever-increasing simplicity? And, in relation to buildings, is not the simplicity of the arch an attraction in itself?

The Origin of Arch Roofs

The idea of the system of construction under review began to take form, in so far as such things may be said to have definite origins, after a visit, 30 years ago, to the Great Arch of Ctesiphon on the banks of the Tigris about 15 miles down-stream from Baghdad. This structure is 112 feet high and nearly 90 feet in span. So far as the Author can determine it is the first column-free rectangular building of importance ever built. Its engineer has apparently been forgotten; the client, the all-conquering Sassanid King Chosroes II; its purpose, the banqueting hall for the King's winter capital. The Romans had not long left London when it was built.

Ctesiphon (*Fig. 2* *) is built of mud bricks, lightly burned and set in Tigris mud. Pitch from the neighbouring lake at Hit and reeds (the straw of Moses?) were used in occasional courses. Standing beneath this vault,

* *Figs 2* to *11* are all photographs and are printed together between pp. 160 and 161.

set in the formless desert, one is struck by a feeling of immensity and by an indefinable impression of stability and permanence—that there is nothing to fall, and that if the local brick robbers had not used the city as a brick quarry to supply the builders of Baghdad, it would remain today "as built." There arises also a feeling of awe because of the incredible daring and the magnitude of such an engineering feat, because of the resource of its creator, craftsman, and engineer, when faced with a seemingly insoluble structural problem in a land which knew neither stone, timber, nor any structural metal. Subsequent check measurements showed it to be a true linear arch, turned (so its soffit appears to indicate) on an artificial mound, trimmed to the curve of the hanging chain, and revetted with the fronds of the date palm.

Justice demands a tribute to the genius of this long-forgotten builder.

Concrete as a Roofing Material

The subject of this Paper is the development of a modified form of Ctesiphon's Arch. It describes research born of a conviction that concrete, the one material now in good supply, is not always being used to its best advantage; and that, as too frequent instances of corrosion indicate, it is sometimes derogated by failure to adopt the most suitable structural forms.

Reinforced concrete is to some extent a marriage of convenience. Concrete, though it clings to its partner with tenacity, and, while things go smoothly protects it from corrosion, gradually withdraws its protection as tensions develop. In spite of reinforcements, concrete commences to "crack", though perhaps invisibly, as soon as its tensile strength is exceeded; and, when it "cracks," damp gets in. Consequently the material has acquired a dubious reputation—"concrete always cracks" has become almost a cliché. This is unfair. Concrete does not "always crack." In pure compression it never "cracks" though it may crush or be affected by tension derived from shear.

Furthermore good concrete, uncracked, does not leak when used as a roof covering laid to a fall. Confidence in this belief has led to the Author being criticized for not coating his roofs with bitumen. Had he done so it would have been impossible to observe their behaviour, and so it was thought better to face accidental leaks so as to find out their causes and then to modify design, or the processes of construction, or both, as necessary to prevent re-occurrence. Readers may conclude from what follows that this course has been justified.

The Introduction and Formation of Corrugations

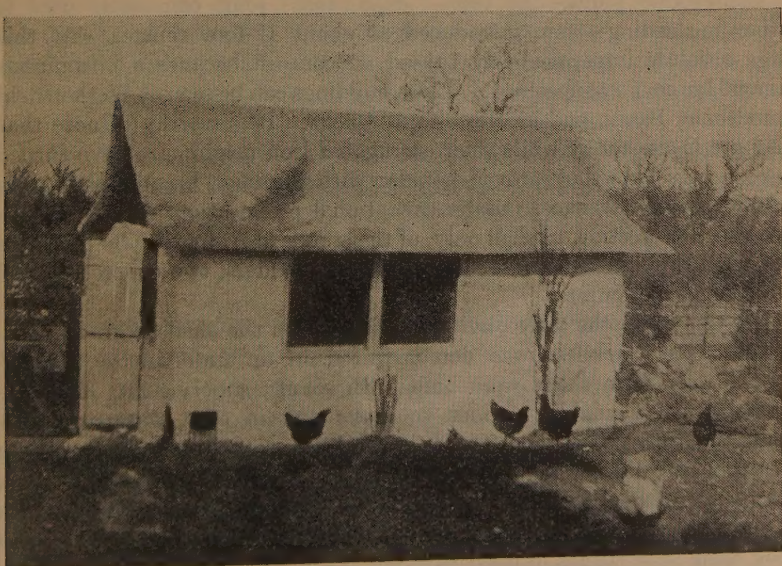
The reason for starting with the catenary arch is simply that in that case the dead-load stresses are purely compressive. Bending moments

due to live load cannot be eliminated, even in an arch ring, and an adequate moment of resistance must be provided. To obtain this by thickening the arch ring, as has been done in the past, is too expensive: by hypothesis, one must economize in steel and thus several other possibilities are also barred. Ribbing the concrete is practicable but the questions of structural efficiency and the cost of centering arise. (See Part 2.)

The use of simple corrugations is one way out and to devise a cheap practical method has been the aim of the Author's work.

For several years various forms of falsework for producing these corrugations were devised. None was simple; all were expensive; none got past the drawing-board stage. Then an earlier experiment came to mind in the shape of the small pitched concrete roof shown in *Fig. 1*. In this case, scaffold poles were set up at the ridge and eaves. The span between eaves was 12 feet. Wires, 3 feet apart, by way of rafters, were stretched tightly over the ridge and fixed at the eaves. Hessian was stretched over this framework and one inch of cement rendering applied. The hessian sagged about 6 inches between the wires and the poles, thus forming a series of inclined buckled plates. When the poles were removed (to the discomfiture of the posse of unbelievers who invariably collect on these occasions—just in case), the structure proved stable and has met with the approval of a long succession of occupiers (of Rhode Island origin) ever since.

Fig. 1



An Early Example of Concrete "Shell" Roof

It may be felt that so unpretentious an experiment as a chicken-house scarcely deserves record. It was, however, a definite attempt—and a successful one—to get depth by deforming a concrete shell by the use of flexible centering, and from it emerged the method of forming corrugations by means of flexible centering now commonly employed. It will be seen from *Figs 3* and *4* that light ribs are set up and braced together, a sheet of light vegetable fabric is stretched over them, and successive coats of rendering applied, by hand or mechanically, until the desired thickness has been built up. When the first coat is applied the fabric sags, thus forming the corrugations, their depth being controlled by the tension on the fabric and the spacing of the ribs.

Experience gained from some Early Structures

The first group of structures of this type, built in 1942–43 onwards, consisted of military huts for personnel, offices, and stores—about fifty buildings in all. The following proportions were adopted :—

Spans : 20, 30, and 40 feet.
Rise/span ratio : $\frac{1}{2}$ approx.
Shell thickness : $\frac{3}{4}$ to $1\frac{1}{2}$ inch.
Width of corrugations : 3, 4, and 6 feet respectively.
Depth of corrugations : approximately $\frac{1}{5}$ of width.
No steel was used and the concrete was applied in two coats.

Contraction joints—the term " expansion " joint commonly used is rather misleading—were introduced at about 12-foot centres. At this stage absolute impermeability caused no concern because a bituminous camouflage coat was specified. These buildings can be seen at Wethersfield Aerodrome, Essex, and at Hermitage, Bucks. It is worthy of note that they are in use today while their corrugated iron neighbours have turned to rust. As huts they proved popular with the men, because they were draughtproof and, except under exceptional provocation, did not sweat. The time for erection, of shell only, of the standard 20-foot-by-36-foot huts was from three to four per week with two scaffolders, two concreters, and three or four labourers.

At this stage the very obvious need to keep the shell at an approximately even thickness was demonstrated in an unfortunate manner. Two 40-foot farm sheds were built with scanty supervision. The contractors, commendably anxious to make certain of a watertight job, made the arches very thick at the crown. Thickness at the springing, it was thought, did not matter because the run-off there was steep. Since there was no steel to take the resultant tension, disaster followed in one case and the other one had to be demolished. In each case the owners took their misfortunes in good part but, as might be expected, the result was a serious set-back in the early struggle for survival that is the common

lot of all non-traditional methods of construction. The only other trouble experienced was in a very long uncompleted shed where four end corrugations failed under a gale which was registered as more than 100 miles per hour.

When the war ceased, and the system began to spread to areas where inspection was impossible, light reinforcements were introduced in order to guard against possible misapplication or misunderstanding of instructions.

In 1948, H.M. Ministry of Works erected a trial 60-foot-span building at their Field Test Unit, Barnet. Field tests were made on this building as follows: the first 12 feet of the building, that is, the first two corrugations, were isolated from the remainder by means of a joint. Straining devices were attached to this section to represent the effect of a wind load on the structure. The inward pressure on the windward half of the arch was reproduced approximately by placing two girders at points across the outer face of the arch and attaching each of these by a pair of ropes passing through the shell by straining devices picketed to the ground inside the building. Wind also produces a suction on the leeward half of the arch, and to provide an equivalent loading for the test a rope-and-pulley system was employed to give the appropriate upward and outward forces; these forces were distributed along the soffit of the arch by means of a suitable framework.

The horizontal and vertical deflexions at eleven points on the arch were measured. A number of tests were carried out, but it is sufficient to mention here that, finally, a loading equivalent to the effect of a 120-miles-per-hour wind was applied; this resulted in a maximum deflexion of only 0·4 inch and there were no signs of cracking anywhere.

This building was reinforced with six $\frac{3}{8}$-inch-diameter rods running transversely over the arch in the crests and valleys of the corrugations, the shell being toughened with 3-inch-by-3-inch 13 S.W.G. wire netting.

Developments

Actual construction has been in hand for about 10 years. The different types so far built include the following :—

(*a*) Military barracks, stores, offices, canteens, etc.

(*b*) Native huts, bee-hive type, 20 feet diameter (E. Africa).

(*c*) Bungalows of various types, a typical example of which is shown in *Fig. 5* (Spain).

(*d*) Church at Bristol. *Figs 6* and *7*. Two more are under construction.

(*e*) Arches, 60 feet in span and of various lengths, in which one or both sides are cut away to provide free access. Thrusts are

taken by A-frames carrying lintel-beams, as in hangars for trainer aircraft and motor transport (Spain) and a large factory (Tanganyika) (*Fig. 10*), or by lintel-slabs supported by columns and tied back to end walls (Eire).

(*f*) Multi-span factories and workshops ranging from double spans (*Figs 8* and *9*) (Great Britain, Eire, Belgian Congo) to a seven-span factory of almost 100,000 square feet covered area (S. Rhodesia) (*Fig. 11*).

(*g*) A wide range of single-span arches used for garages, stores, refugee dwellings, granaries, go-downs, and for all sorts of agricultural purposes.

(*h*) Houses for Africans in various parts.

In all, the system has spread to eighteen countries and the area so far covered is approaching one million square feet.

Summary of Experience

The experience gained by this work may be briefly summarized as follows:

Falsework.—Generally, the type of rib shown in *Fig. 4* has been used, constructed of tubes, angles, or timber. As in all arch work, it is desirable to remove the supports as early as possible and so it is usually practicable to get a turn out of the ribs once a week.

Labour.—In general the work has been carried out by semi-skilled men. In backward parts native labour has proved eminently satisfactory; it has been found that the natives possess considerable skill in applying the rendering.

The cement gun is undoubtedly effective if the scale of the work is sufficient to justify the cost of bringing plant to the job: very-high-quality concrete is produced, which, by virtue of its density, has a comparatively low shrinkage coefficient. For covering large areas the advantages of the gun are strongly indicated, both in the interests of speed and in quality of work. The cement-gun principle, by which the materials are piped in the dry from gun to nozzle, where the water is introduced, is unquestionably preferable to any other method of mechanization, but with large buildings the quantity of material involved appears to call for larger plant giving increased speed of application.

The actual number of man-hours required naturally varies under the widely varying field conditions, but some definite figures for hand-placed shells may serve as a guide. Including erection and removal of centering, on prepared foundations, and working in the autumn on their second building in Aberdeen, one skilled and eight unskilled men erected a 40-foot-by-200-foot shell in 26 working days: or, say, 3 square feet of floor per man-hour. On twin-span work (95 feet overall) in Herts, including columns and valleys, the labour, in mid-winter, came out at just double

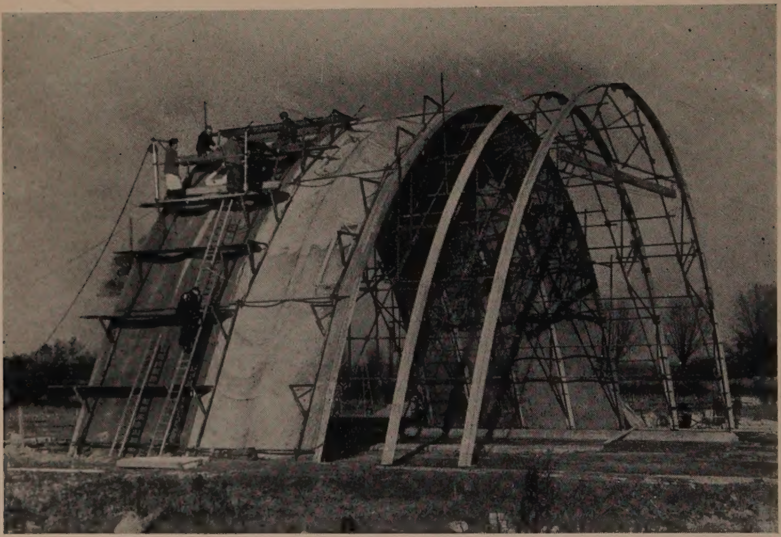
Fig. 2

The Great Arch of Ctesiphon

Fig. 3

Erecting Falsework for a 60-foot Arch
M.O.W. Experimental Building by Field Test Unit at Barnet

Fig. 4



Application of Concrete, using Hessian as Shuttering, on Building shown in *Fig. 3*

Fig. 5

A Spanish Example

Fig. 6

The Church of Christ the King and St. Peter, Bristol

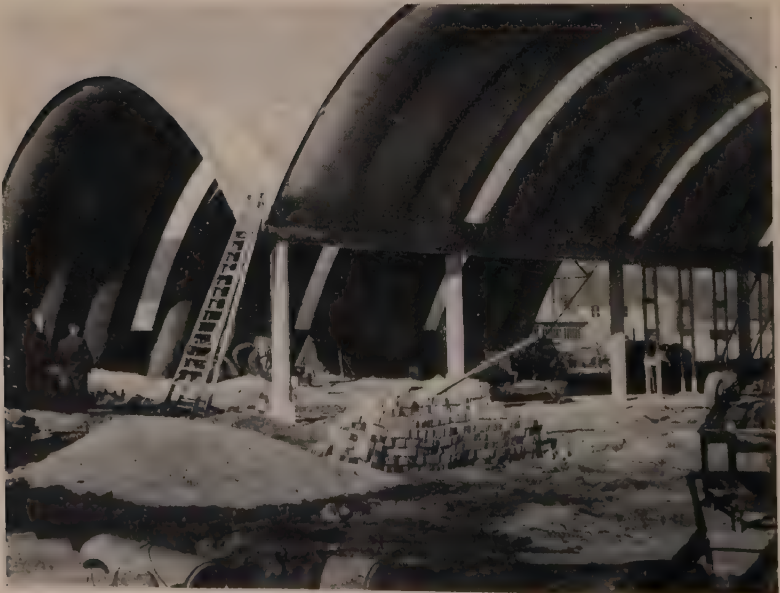
Fig. 7

Interior of the Church shown in *Fig. 6*

Fig. 8

WORKSHOPS AT ROYSTON, HERTFORDSHIRE
(SHOWING ARRANGEMENT FOR SCAFFOLDING)

Fig. 9



WORKSHOPS AT ROYSTON, HERTFORDSHIRE

Fig. 10

Sisal Factory and Dwellings at Mruazi, Tanganyika

Fig. 11

Factory at Umtali, S. Rhodesia
(Floor area 100,000 square feet)

this figure. In each case the figure included glazing in reinforced shells $2\frac{1}{2}$ inches thick.

In very large spans, where it is anticipated that tubular scaffolding would be used, the labour for that item would exceed that for the prefabricated ribs employed on the buildings described above. Taking a 300-foot-span arch, 60 feet high, erection and striking of centering might be taken to represent $\frac{1}{4}$ man-hour per square foot of floor. Four or five inches of concrete would have to be handled but, against that, mechanical aids would be available and the slopes upon which the concrete is to be placed are less serious. Under average European conditions the conclusion has been drawn that such a shell on prepared foundations should be covered by a labour allowance of the order of 1 to 1·5 man-hour per square foot.

Materials.—The thickness of concrete used up to the present is in most cases considerably in excess of minimum requirements and may appear to compare unfavourably in this respect with some contemporary barrel-vault construction. The explanation is that, unlike construction based upon the girder principle, extra shell thickness does not necessitate the provision of an increased modulus of resistance or falsework cost. Careful observations show that once the first inch or so of concrete has been placed the arch has reached a stage when successive coats place no more load on the centering. The bare cost of thickening-up the concrete so as to provide really liberal cover is in itself a small matter and the Author finds it a matter of considerable satisfaction that he is spared the temptation to economize in cover. For instance, a 4-inch-thick shell for big spans is mentioned later. It is made up thus :—

Internal cover	1 inch
Space for a square mesh of $\frac{3}{8}$-inch rods, plus tolerance	$1\frac{1}{2}$,,
External cover	$1\frac{1}{2}$,,
Total .	4 inches

Clearly, without offending any Code of Practice, an inch or more could be taken from this—but to what purpose ?

Ventilation

In some cases complaints were received about the buildings being stuffy. It must be understood that ventilation is more than ever essential, because the buildings are completely airtight. Satisfaction, on the other hand, has been expressed in regard to the natural ventilating properties of the arch when proper provisions are made for the entrance and discharge of the air. In hot climates the absence of all dead air and the comparative loftiness meet with favour. As the shell heats up under strong sun the

intrados transmits heat to the film of air with which it is in contact. The heat transfer is wholly by conduction since the surfaces do not radiate. As this film warms up it becomes lighter and creeps upward; keeping contact, it absorbs more heat until it reaches the outlet and escapes. Remarkable tribute has been paid to the low average temperatures registered, not only in living quarters but in factories and stores used to keep materials which are liable to spontaneous combustion.

Experience has shown two simple ways in which local leakage can take place. If the mortar is applied too wet, or if it is caught by rain and so becomes too wet, there is a tendency for it to drag in places on the steeper slopes. In this way quite short cracks, approximately horizontal, are formed. The larger ones are noticed and closed up by the concreter; the smaller ones are easily missed and have caused anxiety. There being no movement, they are easily closed with a good rub of mortar after a thorough soaking. As a preventive it is recommended that the concreter should go back on his work an hour or more after it is placed and float it over finally. This seals the drag cracks and increases all-round impermeability. It need hardly be mentioned that the wooden float is favoured in preference to steel.

Another caution is necessary. The arch rises and falls and joints between shell and partitions should therefore be made with lime and not cement mortar.

Double shells.—The provision of insulation over and above that provided by the single shell is best effected by constructing a second shell with cavity. The procedure is to lay a line of separators over each corrugation crest, over which is stretched a sheet of hessian followed by rendering. The separators commonly used are bricks or light concrete blocks. For convenience these are set in cement mortar. No connexion is made between the separators and the second skin, so that the differential movement between the two is unrestricted. If desired, the cavity may be packed with insulating material. In one case balsa blocks were used, the local engineer pointing out that the white ants would quickly remove these and so leave the two shells completely free. This, it is suggested, is the first recorded example of friendly co-operation between the parties concerned.

When cold storage or air conditioning is involved two or more cavities can easily be constructed and a very high degree of insulation obtained.

Secondary stresses.—In Part 2 of the Paper the question of secondary stresses is dealt with at considerable length. The following paragraphs deal briefly with their effect upon actual construction.

When a corrugated slab is subjected to bending, stresses develop which tend to make the corrugations open or close. This movement becomes structurally important only when the corrugations are large and, as shown later, may be met either by reinforcing the shell with two layers of

reinforcements or by bracing the corrugations—the method adopted by Mr Freyssinet at Orly. Of the two, the latter is the simpler and more direct method. It consists in introducing horizontal members, in the form of steel tubes or reinforced-concrete members, connecting corrugation crests or valleys. In all large spans it is advisable to place a square mesh of rods in the shell, not so much perhaps to meet any stress that can be clearly defined as to serve as a general toughener. If only concrete, in addition to its other admirable qualities, had more of the properties of leather how nice it would be! The object of this steel is to care for the unexpected and accidental, in particular for the incalculable racking stresses that may be caused by shock (bombs, earthquake, or very exceptional winds).

Next there come the movements caused by shrinkage as the concrete matures, and by temperature change. In the Author's view the former is the more important. Shrinkage across the arch in spans up to 60 feet has not been a difficulty, provided the falsework is removed or eased promptly, leaving the shell free to "breathe." The anticipated effects in spans of more than 60 feet are dealt with in Part 2. Horizontal movement is a different matter because the arch is fixed at springing level. The foundations shrink less and more slowly than the shell and are scarcely affected by temperature.

In small arches transverse joints may be introduced in the crests of each or every second corrugation thus effectively preventing a build-up of tensile stress in excess of that safe for concrete. Observations indicate that contraction from shrinkage is greater than subsequent expansion due to temperature change; consequently it appears that no further precautions need be taken on that account.

In large spans such close spacing of joints is impracticable and a complete break in the shell is desirable. This break may conveniently be made by introducing transverse glazing strips as shown in *Figs 10* and *13*, the toughening steel being relied upon to concentrate movement at these joints.

In a true-arch shell, of uniform thickness, the resultant thrust coincides with the neutral axis of the arch and the conditions to produce pure compression are satisfied. When the shell is subjected to wind or other live loads the line of thrust moves to one side or other but no tension develops in the concrete until the self-weight precompression—"prestress"—is neutralized. This "prestress" (see also Part 2) is a formidable weapon in the hands of the designer who plans to save steel. Owing to our habitual familiarity with beam design its significance tends to be overlooked. Derived from gravity, dependable and determinate, it is the free gift from beneficent nature to those who use the arch. Gravity is destructive to the beam-truss-girder family, but bestows stability upon the arch. Where the ratio of rise to span and the proportions of the corrugations are under the control of the designer, it may be used to neutralize all the main tensile

stresses due to wind, so that the limit of span depends upon the crushing strength of the concrete.

Thus it will be appreciated that, if a working stress of 1,200 lb. per square inch in the concrete be assumed—by no means an extravagant assumption—then an arch, 700 feet in span with 140 feet rise, may be built in plain concrete and the Author invites members to show cause why the shell thickness should exceed $4\frac{1}{2}$ inches.

The Author frankly admits that when he arrived at this conclusion he was startled. It means that the span of these arches can be increased to any extent likely to be desirable from the user's point of view without appreciably increasing the quantity of material per square foot of floor. This is the fundamental difference between the arched and girder types of vaulted roofing.

The Possibilities of Large-Span Concrete Roofs

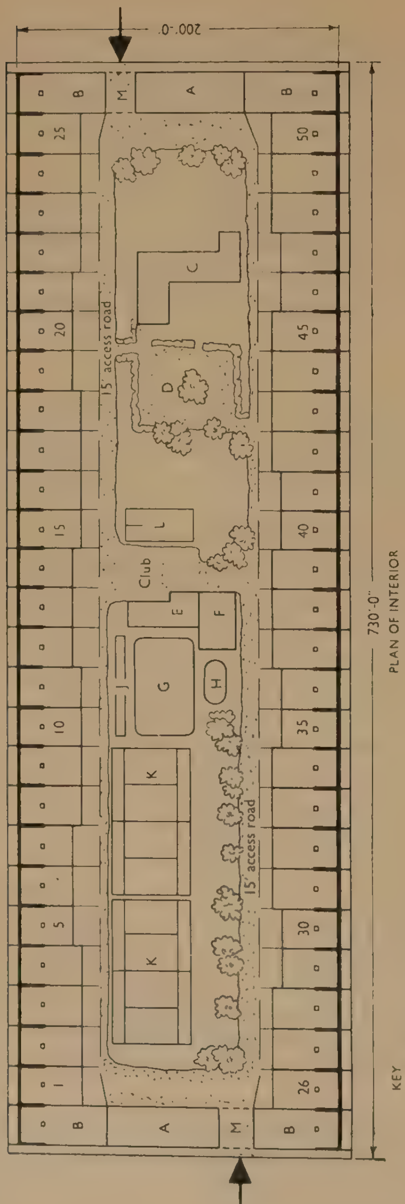
The utility and necessity for large-span arches has been questioned. Is there a genuine demand for such structures, say, 700 feet or more in span ?

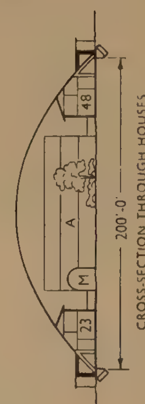
A complete reply to this question cannot be attempted within the limits of this Paper. Such a development in civil engineering practice is new ; structures of this type have never been offered to potential users heretofore and if, as the Author makes bold to assert, they are both practical and economical, the whole problem merits the closest study not only by the engineer, and his counterpart the civil engineering contractor, but by the architect, industrialist, doctor, and economist. If there are to be such buildings, the best way to use them must be learnt. The following paragraphs are intended to suggest the general picture that presents itself.

Considering first the case of an ordinary single-storey industrial building, would single or multi-spans of, say, 200 feet with a rise of about 40 feet facilitate processes of production or the reverse ? That they could compete in cost with all but possibly the lightest cement-asbestos–steel buildings scarcely admits of doubt : space taken up by columns or other supports would be saved, against which, for a few feet at springing, the height would be restricted ; ample headroom would permit the use of heavy lifting tackle ; the marshalling of large vehicles would be facilitated, whilst no permanent obstructions would hamper lay-out. The whole question calls for close co-operation between the architect and the civil engineer.

With regard to the question of climate control, the Author is convinced that there remains much to be done. Remarkable results have been achieved by mechanical, electrical, and kindred scientists in the fields of space, distance, speed, power, sound, light, heat, and so on. Surely it is also possible that, by intelligent application of modern science, the more objectionable conditions of life in extreme climates can be brought under

Figs 12





KEY

1 - 50 Terrace houses
(33 houses with 2 - 3 bedrooms
17 houses with 1 - 2 bedrooms)
A. Bachelors' quarters: 40 bedrooms, mess, etc.
B. Servants' quarters
C. Nursery school
D. Children's playground
E. Club: Open-air bar over library, billiards, etc.
F. Hall and stage
G. Swimming pool
H. Paddling pool
J. Changing cubicles
K. Tennis courts
L. Cricket net
M. Main entrance gates

SUGGESTED LAY-OUT FOR RESIDENTIAL BUILDINGS IN EXTREME CLIMATE

control on a much more extensive scale than at present. For example, in the very hot and often humid climates that prevail in some of the oilfields, the loss from reduced efficiency and the physical discomfort that life in these climates incurs seriously impedes production, to combat which, large sums are spent on air-conditioning individual dwellings and other scattered structures. Would it be sensible to construct large buildings, of, say, 200 to 300 feet span, or larger, and incorporate in them dwellings, schools, and above all, provision for recreation, as suggested in *Figs 12, 13*, and *14*; and to use similar structures for workshops in which all off-site work would be done, and for shopping and business centres? This idea, after all, is but an extension of the age-old Eastern Bazaar.

Fig. 13

Front View of Houses seen from Interior of Main Building

The exposed area of walls and roofs in individual buildings now favoured is considerably greater than would be the case of the proposed corporate building; consequently there would be less area to be insulated and less power required for air conditioning. Concentration would save transmission losses and reduce the cost of all supply services: control would be facilitated and the unpleasant effects caused by mechanical troubles with air conditioning plant minimized. There would appear to be a case worthy of examination on the score of both construction and operation costs. Whether engineers are competent to pronounce upon the general advisability of such a proposal may be open to question; it is certain, however, that they are in a unique position to assess the very costly losses in efficiency and the wastage of personnel that are occasioned by

uncongenial living and working conditions. Further co-operation along these lines between the architect, the ventilating engineer, and the civil engineer is advisable.

As a final speculation, consideration might be given briefly to the

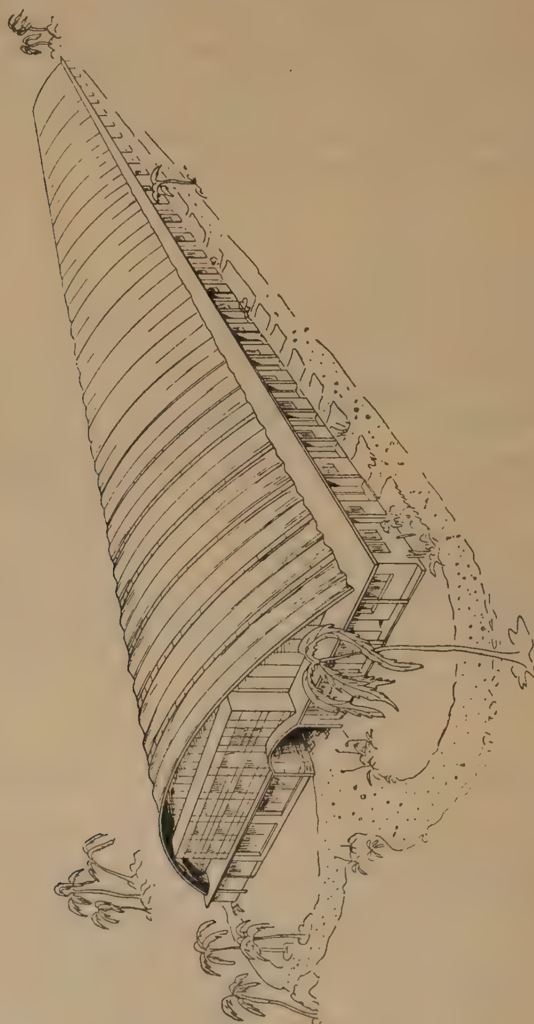


Fig. 14

Aerial View of Main Building

question of air transport and an attempt made to picture some of the developments that hangars—perhaps to be called "archangars" for distinction—of 600 or 700 feet span would make possible.

At one end a door, 200 feet or more in width, leads to a central clearway

running the length of the building. This end provides space to take up and set down passengers, customs, rest rooms, restaurants, administration, and so on. The plane, in tow, passes on and may park on either side or proceed to the far end where are found workshops, stores, and all that is necessary for maintenance. When serviced it returns without let or hindrance, pausing to embark its load, and so proceeds on its way. The whole building is climate-controlled and, by suitable forethought, gives complete protection to the passenger from the discomfort occasioned by the sudden transfer from the miraculous comfort of an air-conditioned and pressurized aircraft into, in extreme cases, tropical heat, sand storm, icy blast, or bitter sleet.

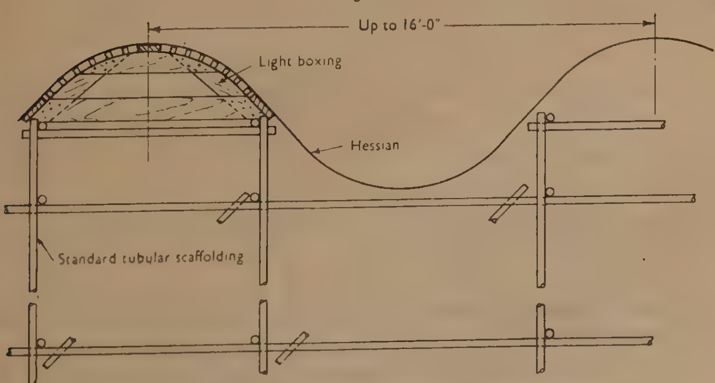
Bearing in mind the achievements of aeronautical engineers in providing for the welfare of the passenger while airborne, shall civil engineers fail to do as well for him when he lands? And if such thoughts are occasioned by the interests of the passenger, what must be said of the claims of the all-important ground staff? Apart from the increased efficiency, provisions such as these would prevent wastage in personnel that alone is worth most serious consideration.

Costs

This subject cannot be left without some mention of cost. What would be involved in the provision of such an "archangar"? To estimate in sterling or dollars for work on unspecified sites is impossible, but broad comparisons may be made.

Comparing first the shell with the runway, the concrete in the shell, though placed mechanically, will cost more per cubic foot than in the runway, because it must be lifted up into position and supported, but it is much less in quantity; reinforcements will be a small item; falsework in tubular scaffolding, say, with light boxing over crests and fabric for the

Fig. 15



TYPICAL SHUTTERING SUITABLE FOR LARGE SPANS

valleys as in *Fig. 15*, would be quickly struck and used many times so that the material expended would be negligible. The cost of shell would definitely be comparable with that of a runway, area for area.

The floor.—The cost would be largely, if not entirely, saved by economies in hardstandings otherwise required.

The doors.—These are a very serious item and, in the hangar, make up a large proportion of hangar cost per machine housed. In the " arc-hangar " described, one door only is required and the number of planes housed is limited only by the length of the structure. In contemporary hangar practice " one plane, one door " is the usual arrangement.

Foundations.—The problem of foundations is rather frightening at first sight, but if concrete stress is limited to, say, 1,200 lb. per square inch in a 4-inch-thick arch, the load on foundations amounts to less than 70,000 lb. per foot run, which is not very serious.

Candid criticism of these proposals is cordially invited, particularly with a view to formulating a complete project for the construction of a prototype corrugated concrete arch of really large span.

Conclusion

This part of the Paper may fittingly close with a brief reference to the tied arch.

Until comparatively lately it has been thought wise to concentrate on the simpler form, springing from ground level, and to await confirmation of the anticipated behaviour of these structures, under varying conditions, before taking the next step.

The introduction of ties raises problems of great interest, especially as spans increase.

Investigations have now reached an advanced stage and proposals for tied arches of large span have been formulated.

It is regretted that a full discussion of this important and entertaining subject cannot be undertaken within the limits of this Paper.

Acknowledgements

Acknowledgement of valuable help in connexion with the development of these structures is due to :—

The late Mr A. D. Delap, M.A., M.I.C.E., the Author's lifetime partner, without whose constant encouragement this work would never have been undertaken.

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Countless other engineers, of both foreman and unskilled grades, without whose interest and loyal co-operation progress would indeed have been tedious.

Part 2

The Theoretical Aspect of Corrugated Concrete Shell Arch Roofs of Catenary Form

by Mr Aston

Stresses in Catenary Arches

A flexible rope of uniform weight, held at its ends and allowed to sag, akes up a form known as a catenary. Conversely an arch of uniform ection and in the form of an inverted catenary curve will be subject to a ure compressive stress, there being no bending moments. The properties f the catenary curve are well known, and the value of the compressive tress at any point on the arch is readily calculated by consideration of tatical equilibrium.

Tables 1 and 2 give the compressive stresses at crown and springing or a range of catenary arches, constructed of concrete with a density of 50 lb. per cubic foot.

These Tables serve to show that the catenary arch has a great initial dvantage over other types of construction, namely, that the dead load roduces only compressive stresses of a low order. Nevertheless this itial compression is of immense value in preventing the development of nsions in the concrete under working conditions.

The statement made as to the absence of bending moments in a

Table 1.—COMPRESSIVE STRESSES AT CROWN OF CONCRETE ARCHES

Rise/span ratio	Span : feet					
	60	100	150	200	300	700
	Compressive stress : lb. per square inch					
$\frac{1}{2}$	19	32	48	64	96	224
$\frac{1}{3}$	27	44	66	88	132	308
$\frac{1}{4}$	34	56	84	112	168	392
$\frac{1}{5}$	41	68	102	136	204	476

TABLE 2.—COMPRESSIVE STRESSES AT SPRINGING OF CONCRETE ARCH

Rise/span ratio	Span: feet					
	60	100	150	200	300	700
	Compressive stress: lb. per square inch					
$\frac{1}{2}$	51	84	126	168	253	591
$\frac{1}{3}$	47	79	118	157	236	551
$\frac{1}{4}$	50	82	123	165	247	577
$\frac{1}{5}$	54	89	134	179	268	626

catenary arch is accurate only under the simplest theoretical conditions. In practice, bending moments arise as a result of :—

(*a*) Wind pressure, snow load, and other imposed loads.
(*b*) Shrinkage of the concrete.
(*c*) Temperature variation.

Bending moments occurring under heading (*a*) can be calculated by considering the structure as an elastic arch, subjected to various possible combinations of imposed loads. Wind pressures may be conveniently taken from the British Standard C.P.3, Chapter V, but the type of structure being considered is not one of the conventional type for which the Code was primarily prepared, and there is room for a divergence of views as to what should be taken as the value of "*h*" (height of the eaves). For buildings of a substantial size it is felt that wind-tunnel tests on a model of the proposed building should be carried out.

Bending moments due to temperature variation (which can be positive or negative) can be calculated from a knowledge of the coefficient of thermal expansion of concrete and the possible variation in temperature of the structure above and below that at the time of construction. The effect of shrinkage of the concrete during the process of setting is equivalent to the effect of a certain fall of temperature. Whilst the coefficient of thermal expansion of concrete is practically constant, and approximates to the value for steel, the coefficient of shrinkage is an uncertain figure, varying with the mix, the amount of water, the method of curing, and other factors.

When designing arches of large span it is necessary to consider in addition the effects of the following factors :—

(*d*) Elastic compression. The slight elastic shortening of the perimeter resulting from the direct compression sets up bending moments similar to those resulting from shrinkage or temperature fall.

(*e*) Movement of foundations (spread or rotation).

(*f*) Deflexion moments. Under the effects already enumerated the arch deflects from its original catenary shape, and the dead load then causes bending moments.

(*g*) Modified-curve moments. It is sometimes advantageous to modify the profile of the arch slightly, thereby setting up a desired initial system of bending moments under the effect of dead load. The object of this procedure is to counteract to some extent the effect of the foregoing factors, and thereby reduce the maximum bending moments.

Arches can be classified as three-pinned, two-pinned, or fixed-end. So ar, arches of the type now being considered have been constructed of the ixed-end type, although the fixity at the springing is generally imperfect. There is no great constructional difficulty, however, in providing hinged onnexions at the springings, and a third hinge could be introduced at he crown if the curve of the arch were modified to take into account the xtra concentration of weight at this point.

In favour of the three-pinned arch is the non-existence of bending noments under headings (*b*) to (*e*) inclusive and the facility and accuracy vith which the bending moments under the remaining headings (*a*), (*f*), nd (*g*) can be computed. Against it are the greater magnitude of the noments under heading (*a*), and certain additional constructional features.

The two-hinged arch suffers from larger moments under heading (*a*) han the fixed-end arch, but the somewhat uncertain moments from eadings (*b*) to (*e*) are much less. It is therefore felt that there may be nuch to favour the use of pinned connexions at the springings for arches f perhaps 150 feet or more in span.

Choice of Size and Shape of Corrugations

The structural purpose of the corrugation is to resist the bending noments which arise from the causes just enumerated.

The process of arriving at a suitable form of corrugation is one of trial nd error. The system of constructing the shell, described elsewhere in nis Paper, allows great latitude in the pitch, depth, and thickness of the orrugations, but at the same time practical considerations of utmost conomy favour thin shells, moderate or small pitch, and moderate or nall depth/pitch ratio.

At first glance it might appear that it is essential only to increase the epth of corrugations until a sufficient section modulus is obtained, but nis is not necessarily so. Except in the case of the three-pinned arch, ne bending moments due to temperature, shrinkage, elastic shortening, nd yield of foundations increase in proportion to the stiffness of the arch, that increased depth of corrugation results in an increase in the stresses

from these sources. It is therefore found that there is a depth of corrugation in relation to the size of the arch, at which the main bending stress will be lowest. The value of the main bending stress is, however, not the only consideration, and it is better to use a corrugation considerably smaller than the value just mentioned. This involves a small percentage increase in the main stresses but a very great reduction in the secondary stresses, as will be seen later. As a rough indication, the best depth of corrugation lies between one-twelfth and one-twentieth of the rise of the arch.

The pitch is generally about four times the depth of the corrugations. A greater pitch/depth ratio facilitates placing the concrete by reducing the side slopes, but results in increased secondary bending moments in the shell.

With regard to the thickness of the shell, the aim is to use the thinnest shell which will prove durable and provide adequate cover for the reinforcement, namely about $2\frac{1}{2}$ inches. It is interesting to note that, whilst from practical considerations such a thickness has been generally used for small-span arches up to about 60 feet, yet from the point of view of strength this thickness appears to be sufficient for larger spans, up to perhaps 150 feet. Further increase of span is possible with a less-than-proportionate increase of shell thickness.

The Stresses in a Corrugated Concrete Shell Roof

Since the terms "main stresses" and "secondary stresses" are used in a variety of senses in structural engineering, it will be well to define their use in this Paper. The main stresses are those resulting directly from bending moments in the arch, under all the headings previously mentioned. They consist of circumferential extensional forces in the crests and troughs of the corrugations, together with the shear forces necessary to produce equilibrium. The secondary stresses are defined as those produced by the radial components of the main stresses. They take the form of bending moments in the shell, tending to distort the shape of the corrugations.

The Main Stresses

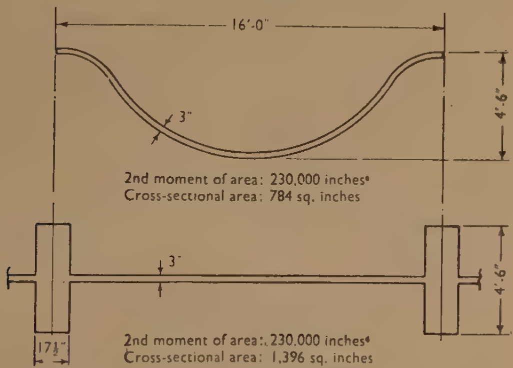
Mention has already been made of the main bending moments which arise in the arch from numerous causes, and the structural object of the corrugated form of the shell is to provide a cross-section with a great moment of resistance. The corrugated form is stronger than a rectangular beam of the same cross-sectional area and overall depth.

To make a comparison with another system of shell arch roof, having the same width of bay and the same overall depth, as shown in *Fig. 16*, the shell-and-rib type of construction uses 78 per cent greater sectional area to give the same moment of inertia as the corrugated shell.

The corrugated section also provides ample width at the crests and valleys for placing the main steel so as to give the maximum lever arm.

In all designs so far prepared for arches of the order of 300 feet span, very little main steel is required. In fact, under the severest combination of bending moments there is rarely any tension developed except within a few feet of the springing when these are restrained against rotation. For example, referring to Table 1, a 300-foot-span-by-60-foot-rise arch will have a dead-load compressive stress at the crown of 204 lb. per square inch. The maximum crown bending moment produces an extreme fibre bending stress of barely this figure, even assuming an unreinforced section,

Figs 16



COMPARISON OF CONCRETE SECTIONS TO SHOW THE ADVANTAGE OF THE CORRUGATED FORM

so that there will be no resultant tension, and the maximum resultant compression will be about 400 lb. per square inch.

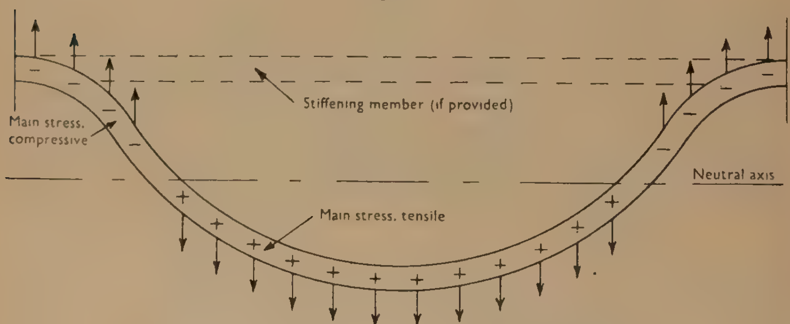
Apart from practical considerations, the main reinforcement is therefore required only to provide for a suitable load factor. For example, using a load factor of 2, the reinforcement in the example quoted must be sufficient to carry a possible tension in the concrete of about 200 lb. per square inch. The end fixing moments at the springing may be relatively high, but even so they will only produce tension in the concrete for a distance measured along the perimeter of the arch of about 10 feet to 15 feet from the abutments. Additional reinforcements and, if required, additional shell thickness can easily be applied in this region.

The main shear stresses, resulting from the variation of the bending moment around the arch, are always of small magnitude. In a 300-foot span the maximum value of the shear stress is of the order of 20 lb. per square inch. It is not, therefore, necessary to provide any shear reinforcement.

Secondary Stresses

Considering the main stresses due to a positive bending moment in the arch, but ignoring for the moment the dead-weight compressive stress, there will be a tensile stress in the troughs of the corrugations and compression in the crests. Since this tensile stress in the trough is acting around the curve of the arch, it has a radial component inwards. Likewise the compressive stress in the crests has a radial component outwards. These inward and outward forces are in balance over the width of one corrugation, but produce a transverse bending moment in the shell, tending to deepen the corrugation. Conversely, a negative main bending moment in the arch tends to cause a flattening of the corrugations. (See *Fig. 17.*)

Fig. 17



RADIAL COMPONENTS OF THE MAIN STRESSES

These secondary moments in the corrugated shell are an important consideration in the design from the outset, since they are a governing factor in the choice of shape and size of corrugation and thickness of shell.

The secondary stresses resulting from these moments can be kept at a moderate figure by four main methods which will now be examined in some detail.

(1) *Use of small-pitch corrugations.*—Other things being equal, halving the width of the corrugation quarters the secondary stresses. But it must be borne in mind that the section modulus of the corrugation is at the same time reduced, thus increasing the main stresses and hence the secondary stresses. Weighing up these two opposing considerations, together with various practical aspects of construction, the best size of corrugation generally lies within the limits indicated earlier.

(2) *Increase of shell thickness.*—This is, from the economic point of view, a last resort, and it is only in very-large-span arches that the minimum thickness of $2\frac{1}{2}$ inches is insufficient.

(3) *Use of two layers of reinforcement.*—In cases where the secondary moments are high, a better use of the secondary reinforcement is obtained by dividing it into two layers, near the upper and lower surfaces of the shell respectively. It is practicable to do this only when the thickness of the shell is 4 inches or more.

(4) *Use of stiffeners.*—To prevent the corrugations from deepening or flattening, stiffening members may be added between the crests of the corrugations, or between the troughs, or a combination of both. Such members must be capable of carrying both tensile and compressive forces, and may be constructed of reinforced concrete (either precast or cast in situ), prestressed concrete, or steel. Their use greatly reduces the secondary stresses, and in a typical example the secondary stresses are reduced to only one-eighth of their value in the unstiffened corrugation.

It may be mentioned at this point that normally the length of roof between expansion joints will comprise perhaps five or six continuous corrugations. The inner corrugations are restrained greatly by their neighbours, and suffer less from secondary stresses than the corrugations adjacent to an expansion joint. It is therefore possible to reduce the secondary reinforcement and even the shell thickness for the intermediate corrugations of a length of roof. Should stiffeners be used, it may be necessary to employ them only across corrugations adjacent to an expansion joint.

Effect of Corrugation Deformation on Main Stresses

In the case of untied corrugations, an important further point is that the arch is more flexible as a result of the ability of the corrugations to distort. In other words, the effective moment of inertia of the corrugation is less than that calculated simply from the geometry of the cross-section. The ratio $\dfrac{\text{effective } I}{\text{geometrical } I}$ varies both with the proportions of the cross-section of the corrugation and with the radius of curvature of the arch at the cross-section considered.

To give some indication of its importance, this ratio may have a value of $\frac{1}{2}$ or even less. Whilst the main bending moments due to wind and snow loads are not affected by this phenomenon, those due to temperature, shrinkage, elastic compression, etc., are reduced in proportion to the effective moment of inertia. It is therefore necessary to recalculate the main bending stresses in the arch, taking into account this distortion of the corrugation.

Discussion of the Present Method of Structural Design

The process which has been used so far for calculating the stresses in the type of shell roof under consideration is a logical one of four main stages, as follows:

(1) The structure is considered as a simple arch of constant cross-section, of catenary profile, with abutments having a certain degree of fixity (or pinned if so constructed). Bending-moment diagrams are produced for this arch by using the well known elastic principle.

(2) The corrugated cross-section of the arch is next considered, and its moment of resistance as a reinforced-concrete member is calculated. Hence the main bending stresses and shear stresses are computed.

(3) The radial components of the main stresses are calculated, and the corrugated cross-section is again considered, this time as a shell subject to bending moments within its thickness. Hence the so-called secondary stresses are calculated.

(4) The effects of corrugation distortion, occurring as a result of (3), are calculated in relation to the main stresses, and all previous calculations are appropriately modified.

It is felt that this approach to the design is more convincing to the non-mathematically minded engineer than an attempt at analytical calculations, even if such should prove practicable. The meaning of each step in the calculation is readily visualized, and the errors arising either in the arithmetic or from the elastic assumptions can be estimated. Furthermore, its extension to take account of plasticity and theory of rupture appears to be straightforward.

In the present early stages of the development of this type of structure for very large spans it has only been practicable to carry out sufficient calculations to show that stresses in proposed designs are everywhere well within the accepted limits, even under the most adverse conditions and assumptions. This assures a very conservative design, and convinces the Authors that when more refined methods of analysis are developed, with the corroboration of practical tests, even more remarkable economies in the cost of these roofs will be possible.

Particular points which appear to require further investigations are :—

(1) In relation to the main bending moments, the ascertainment of more exact data regarding the fixity of the foundations, the amount of shrinkage to be allowed for in the proposed system of placing the concrete, and the distribution of wind pressure over the roof.

(2) The amount of corrugation distortion which will actually occur, bearing in mind the stiffening effect of the adjacent corrugations and the abutments. So far, calculations have been made to cover all conditions from complete fixity to complete freedom, unless circumstances clearly indicate otherwise.

(3) The desirability of extensive use of stiffeners to prevent distortion of the corrugations and reduce secondary stresses.

(4) Whether the theoretical advantages of the two-pinned, and

especially the three-pinned arches, are such as to outweigh the slight additional constructional complications.

(5) The possibility of local buckling occurring in the thin shells, especially near the end of a section of the building, and to what extent the safe working stress should be reduced to take this into account.

The Paper is accompanied by eleven photographs and ten sheets of drawings, from which the half-tone page plates and the Figures in the text have been prepared, and by the following Appendix.

APPENDIX

THE DESIGN OF PROPOSED AIRCRAFT HANGARS AT LARA, VICTORIA, FOR THE COMMONWEALTH OF AUSTRALIA

The principles of design which have been discussed in general terms in Part 2 of this Paper will be more readily understood by reference to a particular example.

A preliminary design has been prepared for corrugated concrete shell hangars for Lara, Australia, as an alternative to the original proposal of steel hangars clad with corrugated sheeting. Tenders have been submitted and are under consideration.

Five hangars are required, all of identical arch profile, and having lengths respectively: one of 306 feet, two of 120 feet, and two of 200 feet. Great economy in falsework is therefore possible since it is intended to supply only sufficient for the erection of part of a barrel at a time and to re-use this no less than fifteen times. Stress calculations were required to be made on the following basis:—

(*a*) Wind load generally in accordance with B.S. 449, but with certain specified increases; wind velocity 75 miles per hour.

(*b*) Superimposed load of 5 lb. per square foot of supported area, as an alternative to the wind load. (Actually this superimposed load does not influence the design.)

(*c*) No snow load.

(*d*) Temperature variation plus or minus 40° F.

Figs 18 show the clearance that was called for both through the doors and throughout the length of the hangars. The profile of arch which was finally selected to give this clearance has a span of 310 feet and a rise of 64 feet, measured to the neutral axis. *Figs 18* also show a part of the side elevation.

In choosing this profile the following matters have to be considered.

(*a*) A flatter arch has the advantages of easier placing of concrete on the slopes, less height for raising materials and erecting falsework, less sheeting required to clad the ends of the buildings above the doorways, and less effect of wind.

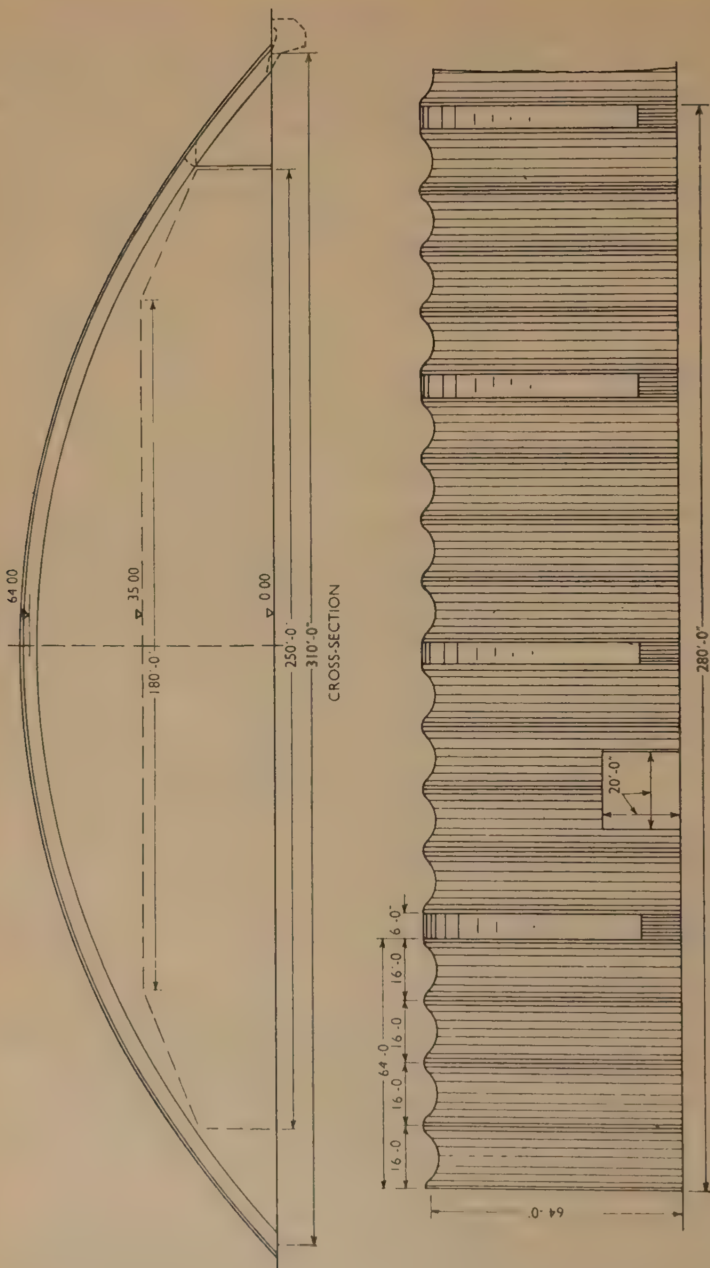
(*b*) An arch of greater rise has the advantages of reducing the span, easing the solution of the temperature and shrinkage problems, and reducing the size of the foundations on account both of the reduction of the thrust at the springing and its inclination to the horizontal.

Main Stresses

The direct compression due to the dead weight of the arch amounts to 210 lb. per square inch at the crown, increasing to 275 lb. per square inch at the springing.

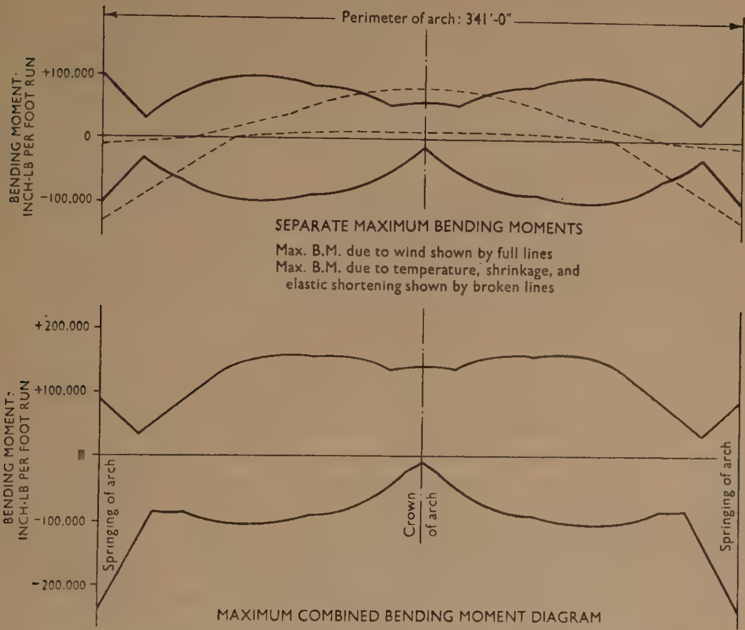
The maximum bending moments are shown in *Figs 19*. In the upper diagram are shown curves for the maximum bending moments under wind load (full line) and under the effect of temperature, shrinkage, and elastic shortening (broken line). The lower

Figs 18



PROPOSED HANGARS FOR COMMONWEALTH OF AUSTRALIA AT LARA, VICTORIA

Figs 19



MAXIMUM-BENDING-MOMENT DIAGRAMS FOR CORRUGATED CONCRETE ARCH ROOF, 310 FEET SPAN, FOR HANGAR AT LARA, AUSTRALIA

diagram in the same Figure shows the maximum combined bending moment from all sources (*a*) to (*f*) listed earlier.

It might be mentioned that, in the absence of definite information as to site conditions, these preliminary calculations have been carried out to cover any degree of end fixity against rotation between 25 and 75 per cent.

It will be noted that over the central portion of the arch the positive bending moment predominates. In order to make the positive and negative moments more nearly equal it is proposed to modify the profile of the arch slightly, decreasing the radius of curvature at the crown and increasing it elsewhere. A suitable choice of profile, which, incidentally, differs from a true catenary by not more than 6 inches at any point, effects a marked reduction in the value of the positive bending moment. A further practical point is that this modified profile can be represented as circular curves of only two different radii, thus simplifying the construction of the falsework.

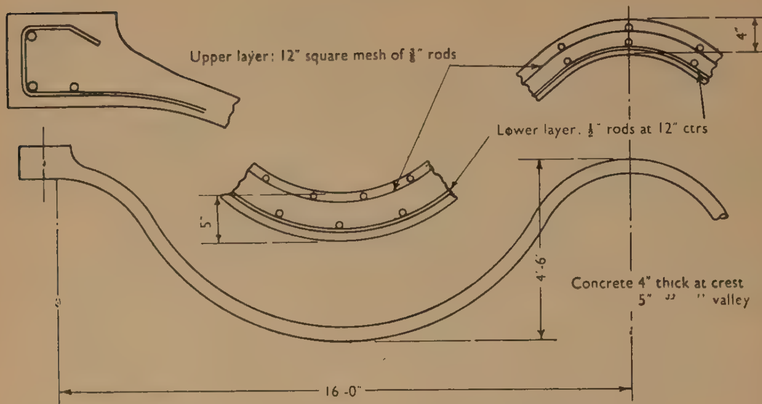
The proposed cross-section of the corrugations is shown in *Fig. 20*.

Apart from the portions of the arch within 15 feet of the springing, no main tensile stress is developed in the arch under the combined effect of dead weight and maximum bending moment. The maximum main compressive stress is 560 lb. per square inch in the crests and 392 lb. per square inch in the troughs. At the springing, taking the case of the maximum degrees of end fixity, it is possible for a tension of 205 lb. per square inch to develop in the crests, and sufficient reinforcement is provided to resist this. The maximum compressive stress at the springing is 620 lb. per square inch.

Secondary Stresses

The highest secondary moments occur in the troughs of the corrugations, and for this reason the thickness of the shell is increased in this region, and a double layer of reinforcing mesh is provided.

Figs 20



HANGAR FOR LARA. SECTION THROUGH CORRUGATION

The maximum shell moment is 910 lb.-inches per inch width, producing a compressive bending stress in the concrete of 420 lb. per square inch and requiring reinforcement of ⅜-inch-diameter rods at 9-inch centres.

It is probable that, in the final design, stiffeners will be used to prevent deformation of the corrugations. These stiffeners will reduce greatly the secondary stresses, and permit a reduction in shell thickness and quantity of reinforcement.

Reinforcement provided

In addition to the reinforcement provided to resist all calculated tensile stresses, there is a minimum quantity of steel provided throughout the whole of the shell consisting of ½-inch-diameter rods at 12-inch centres. This is intended to act as a toughening mesh to withstand the effects of local temperature and shrinkage variations and to ensure an ample margin of safety in view of the novelty of this type of construction in such a large span.

This grid of reinforcement also forms an essential item in the process of construction, since it acts as falsework for the placing of the gunned concrete.

Quantities of Material

The following figures will afford an interesting comparison with other types of concrete construction. The quantities of steel and concrete cover everything in the structure above ground level, excluding only the end-door construction.

The shell thickness ranges from 4 inches to 5 inches, the average being 4½ inches.

The total quantity of concrete in the building, in cubic feet, divided by the floor area covered, gives an average concrete thickness of 5 inches.

The weight of steel reinforcement provided is 2 lb. per square foot of floor area.

Discussion

Mr C. V. Blumfield said that the Paper had presented many new ideas. He had been particularly impressed by the sketches of the shape of things to come. Whether all the ideas were desirable from the practical living point of view he did not know, but they certainly provided a very interesting speculation.

Snow load was mentioned in the Paper, but it would be interesting to

know what exact figure had been taken for snow load because the code of practice was not very clear on the point. Secondly, was snow load taken over portions of the arch? In the Paper there was an account of one shell where the thickness of the crown was made rather more than had originally been intended, and it seemed that the effect of snow would be rather similar if there were a snow cap. Most of the arches illustrated were fairly steep so that the snow danger was not likely to be so great, although there was an illustration of the proposed hanger at Lara for which the arch was quite flat—although that particular building, being in Australia, would not be affected by snow. If arches were built in places where snow was a possibility, were extra precautions taken and were there amendments to the section and rise compared with the width?

Were any figures available for the amount of vertical deflexion? When an arch was fixed-ended, expansion and contraction were bound to cause lengthening and shortening of the arc, and the only way in which it could move was up and down. Were any data available which measured that effect?

When the system of construction first became the subject of comment, Mr Blumfield had been under the impression that the hessian was the only reinforcement and he had been surprised, on reading the Paper, to realize that there was steel reinforcement. Presumably, for small shells, where it could be shown that the tension stresses were within the limits of the tension stresses of the concrete, hessian could be used without steel, but for the larger shells steel was required. It would be interesting to know where the limit of no steel was taken.

The difficulty in popularizing and using a shell arch of Ctesiphon type was that, whilst it was a perfectly good structure for certain purposes—for storage and suchlike—it was very largely limited to such purposes. The very large volume of air required for air conditioning, heating, and ventilation made it rather a difficult problem for ordinary factory construction.

Of course, the great height made artificial lighting a difficult problem. Those problems could be overcome. Nevertheless, in Mr Blumfield's view buildings of that type were limited to the shed building, where undoubtedly they were a great success.

Although the cost was mentioned in the Paper, there were no figures which gave clearly the cost per unit of plan area and it would be interesting to have further figures. The cost of the shell itself might be light, but the considerable perimeter compared with the floor area might increase the cost a good deal.

Referring again to the question of snow, Mr Blumfield said he had mentioned it, in particular, because he recalled seeing a large-span arch collapse at Southampton because of snow. It was a span of about 120 feet, and undoubtedly the snow had imposed bending moments which the arch was not capable of carrying. It was not one of Mr Waller's roofs nor one of Mr Blumfield's!

Mr J. M. Watson observed that the problem of enclosing space cheaply was always of interest to the Air Ministry, particularly in connexion with aircraft hangars, but there were many complications—for example, the door openings which had to be provided at each end of the hangar. The Authors had shown that there might nowadays be door openings of the order of 250 feet span; that requirement would have to be met, at both ends, to allow access to any of the aircraft housed in the hangar and to enable the hangar to be emptied quickly if necessary. Workshops were required, situated conveniently to the hangar and preferably opening direct into the hangar space. The hangar must be adequately heated and lit because it was not normally used for storing aircraft but for aircraft under repair; if necessary, the staff worked on those aircraft by day and night.

The Authors' proposal to adopt the catenary arch as a basis would, Mr Watson thought, appeal to all engineers as an essentially economic approach. In itself, of course, it was a pre-stressed concrete design and it could be said to be as up-to-date as it was ancient in conception. In expanding the conception from moderate spans to major spans, however, a number of problems would arise.

Mr Watson referred to Mr Waller's comment on the remark that concrete was a material which always cracked. Mr Watson was quite certain that concrete often did crack, and most of the reasons for that were known; there might be unforeseen stresses which were very difficult to calculate and which had not been fully taken into account; insufficient allowance might have been made for contraction and expansion by expansion joints; bad workmanship could cause cracking, for example, porosity or displacement of the reinforcement could cause rusting, followed by cracking. But there was equally no doubt that properly designed and properly placed concrete did not crack. The difficulties in producing it were well known.

On p. 163 there was a statement which appeared to suggest that hangar surfaces did not radiate internally. From personal experience, Mr Watson knew that a concrete building which was standing in the sun radiated no small amount of heat internally. Had he misunderstood the Paper or would the Authors explain their meaning?

On the same page the Authors had referred to double shells, and there was no doubt that a double shell of some sort would be necessary if a satisfactory standard of insulation were to be achieved compatible with heating the hangar. In the case of the double shell described by the Author, the external shell became the stressed shell, and had to take the wind and snow loads; it must therefore contain the reinforcement. Mr Watson could foresee some difficulty about its construction on top of the first shell and possibly about the placing of the reinforcement. It would have to be shuttered over the crown of the corrugations and the shuttering would be left in situ, which would offset some of the saving on shuttering

underneath the arch. Had the Authors considered whether it was practicable to hang hessian internally and gun it from underneath to provide the second shell and an air space ?

Heating the hangar was a problem, because there was a tremendous amount of air space to be heated before the heat reached where it was wanted—below about 20 feet from the ground. That was a disadvantage of the ideas put forward by the Authors, in the Appendix, for a 300-foot hangar, where the crown was 64 feet from the ground.

On the question of the shell itself, Mr Watson asked about unsymmetrical loading. Wind stresses had been dealt with and Mr Blumfield had spoken of snow stresses. The code of practice called for 15 lb. per square foot over the crown, tapering off to nothing at 75 degrees slope, which would impose a flattening stress. Would the Authors comment on that ?

There was no mention in the Paper of the wind load on the ends of the building, but with a hangar of 300 feet span, 64 feet high, there were very considerable wind reactions to be taken into account with an end wind. It appeared that that might involve a separate framework from the arch, for Mr Watson doubted whether the arch could carry that stress. The original Ctesiphon arch seemed to have very massive ends, as though end-wind reactions had then been dealt with in that way.

Hangars must have workshops adjacent, and the workshops needed wide openings into the hangars from the curved wall. The openings would be of the order of 20 feet high and might extend the full length of the hangar. Could the Authors cope with that requirement, which was quite definite ? The indications from one of the illustrations were that the arch reactions would be carried on a portal frame, and Mr Watson asked for some information about the design of the frame.

Another point was that lifting tackle, if required, would need a separate frame, for the height of the hangar would create difficulties.

Mr Watson said the construction of arches up to 60 feet span had been successful, but an extension to spans of the order of 300 feet would, in his opinion, introduce difficulties of dimension—the difficulty of placing the concrete, in the first instance. It would presumably have to be placed symmetrically around the shell. One part of the shell could be in the heat of the sun and the other side exposed to a cool wind and in the shade. On the following day, when the second coat was added, the situation might be exactly the reverse. There was thus a danger of stresses being set up between successive layers of concrete. Had the Authors given any thought to that ? The curing of the concrete would be a difficult problem because in a large building, of 300 feet span, it would be difficult to prevent severe cracking in a drying wind.

The catenary principle was, however, essentially attractive. The Authors must have found much satisfaction in their development of it, but to extend from 60 feet span to 300 feet span in one step was a big

jump and the development might, perhaps, proceed by intermediate stages with advantage.

Major-General A. G. B. Buchanan remarked that he had first been made aware of the Ctesiphon form of construction as propounded by the Authors in the very early stages of its development; that was about 8 years previously.

It was interesting to consider what might have happened if the development had taken place a few years earlier. Many members would recall that one of the principal engineering tasks of the last war had been the provision of storage, and one of the greatest difficulties had been the shortage of steel. They would also remember that from about 1942 onwards the Romney Hut, which was a great advance on all its predecessors, had been used practically exclusively in Great Britain to provide storage. The Romney Hut needed about 6·65 lb. of steel per square foot covered. The Ctesiphon arch, on the other hand, needed less than 1 lb. per square foot for the normal kind of 60-foot span, which was an immediate saving of 5½ lb. per square foot. For the American Forces alone the War Office had provided more than 6,000,000 square feet of new storage in Britain and a simple calculation would show that, had the Ctesiphon arch been developed at the time and had it been used, more than 14,500 tons of steel would have been saved on that one item. In the Middle East Bases 24 million square feet of new storage had been provided, involving about 60,000 tons of steel which could have been saved—and all that steel had had to be carried long distances.

Another point which interested the military engineer deeply was that of camouflage. The arch, with its smooth slope, and with the absence of shadow, would be a paradise for a camouflage officer.

General Buchanan said that *Figs. 12*, *13*, and *14* were particularly interesting to him because the problem of cost was at first sight rather frightening. He recalled that during the war a vast network of tunnels had been created in the Rock at Gibraltar, and, to their surprise, it had been found that the cost of tunnelling, plus the cost of putting up a light structure inside, was about equal to the cost of a building of the same capacity outside. That might sound fantastic; and it might also sound fantastic to build a large arch and to put buildings underneath it, but it should be borne in mind that a very light construction could be used inside.

He found it curious that the Ctesiphon system of construction had not become popular in Great Britain, although in the Empire, the Middle East, and the Colonies it was being increasingly used. There were three reasons, he thought, for the different attitude in Britain. First, there was the innate conservatism towards anything new. Secondly, there was an old saying that a prophet was without honour in his own country (although that did not apply to Mr Waller). Thirdly, there was the rather unusual shape.

There were plenty of places where the arch would be suitable, and he hoped there would be a large number of examples in Britain before long.

Mr P. de H. Hall said there was no doubt that the catenary curve was fundamentally the right form for an arch if it were to be built of a material which lacked tensile strength. Having accepted that, there was little in the Paper to criticize, except in detail. Speaking as a constructional engineer who had long ago given up the tee-square for gum boots, and who had forgotten all about design, Mr Hall said he hesitated to make any criticism of Part 2 of the Paper. He had read it with great interest and it had seemed to him that all the likely loads had been dealt with.

Figs 12 illustrated an interesting idea of housing a village under one roof. It was suggested that it would be useful for eastern countries which suffered extreme climates. But Mr Hall felt there might be difficulties. Some years ago he had been dealing with the problem in the East, where normally flat roofs were used, and had gone to a lot of trouble to insulate them so as to provide cool conditions inside, but without much success. It had been suggested to him that an umbrella over the top would result in shade temperature. It had sounded all right, and had been tried by placing steel trusses and corrugated iron on top of the flat roof, and shutting it all in with gable ends to keep out the wind (for the rainfall could be very heavy in that part of the world). But the result was not a success. The sun beat on the corrugated iron and stewed the air inside with the result that the position was no better than before. The only good result had been to make the flat roof more waterproof—which was not the original object of the experiment!

If Mr Waller tried to put a village under one arch he would have to keep the ends open and orient it in such a way that the sun passed over it transversely rather than lengthwise, and so that any prevailing wind would blow straight through the tunnel. If the air were kept moving shade temperature might be attained, but not otherwise.

Mr E. H. Lewis-Dale, before turning his attention to aircraft hangars, recalled a comment he had heard regarding the church illustrated in *Fig. 6*. A friend had said it looked as though it had been turned out of a jelly-mould; Mr Lewis-Dale, however, thought that some might consider it an improvement on a much-talked-of example of modern ecclesiastical architecture!

Necessarily, in an arch sprung straight from the ground, there was some space at the side of the hangar which could not be used for aircraft storage, and it would be possible to use the space for annexes. That might be the Authors' intention. But the economic use of that space within the hangar must limit fairly precisely the size of the annexes, and there was a possibility of losing some flexibility. If flexibility were lost in that way, there arose the perpetual problem of the economical use of space at airports.

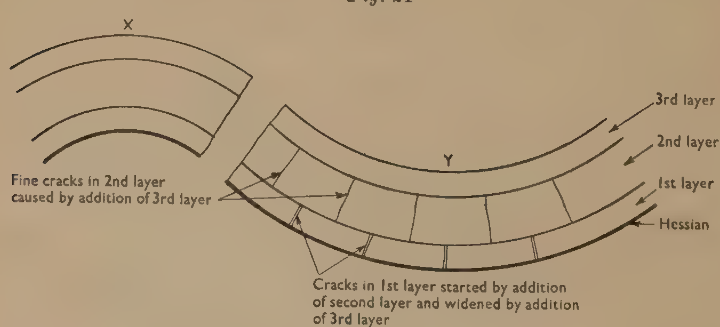
Mr Lewis-Dale's experience had been that, apart from the very large airports, there was never as much room for a building as there appeared to be at first sight. Frequently, the choice of an arch had to be modified by limitation of space. Perhaps that difficulty could be overcome, since designers were not necessarily limited to square hangars.

The Authors had asked for criticism of the "archangar," and his view was that in future there might be a reason to cover the passengers entirely. That was based on the present terminal facilities at Renfrew airport which, by a miraculous accident, were in a hangar which had a curved roof—although nothing like the Authors' arch. Various methods of bringing aircraft into position so that goods and passengers could be loaded or unloaded had already been described in a Paper presented to the Airport Division.[1] Mr Lewis-Dale did not think it was wholly necessary to bring an aircraft back into a large-span hangar for such purposes.

There were, however, two types of railway station—one with an open platform and one with a big arch covering—so that there might be a future for the suggestion put forward. He did not think it likely to happen for a long time, because, in the present stage of airport development in Great Britain, he was certain that there was not enough room.

Dr T. P. O'Sullivan, referring to *Fig. 21*, said that the general policy seemed to be that X would be on a fixed platform and Y would be on a hessian mat. If the concrete was made up in more than one layer, what

Fig. 21



would be the result on the hessian mat? The application of the first layer would result in a certain stretching of the mat. Would not a further addition of concrete result in an additional stretching which would cause cracking in the concrete at Y? It appeared that in that method of construction in several layers only the last layer was fully effective, or as effective as could be wished.

Perhaps the Authors would agree that, with such large spans, it might

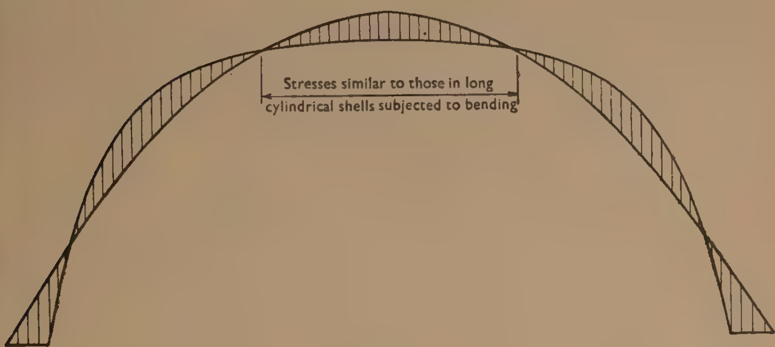
[1] W. J. Cozens and J. A. Glen, "The Influence of Ground Handling Facilities in Relation to Airport Lay-out." Airport Paper No. 13, Instn Civ. Engrs, 1950.

be desirable to do it all in one layer. What was the limitation in the use of the hessian mats in large spans? And what was the effect of secondary stresses?

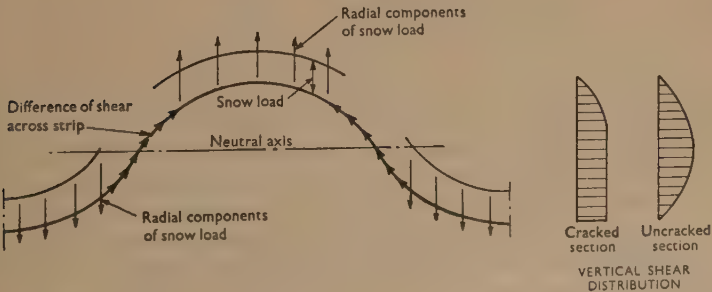
While it was known that a number of arches had been built in the smaller spans, Dr O'Sullivan said that it was also known that none was yet as large as 700 feet. Could the Authors give a summary of data for the larger spans built so far, including the thickness used and the modules?

Professor A. L. L. Baker said that he had not had time fully to study the theory of the arches, but it had occurred to him that the Paper

Fig. 22



Figs 23

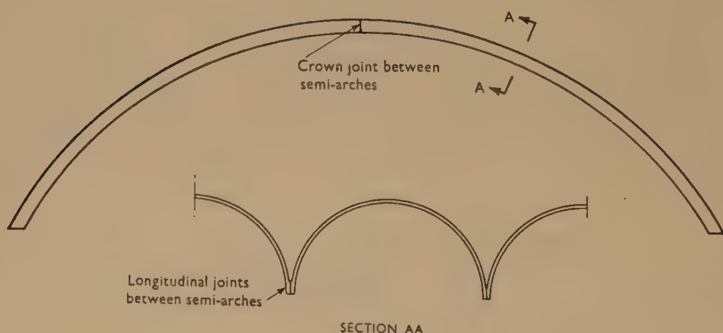


did not mention shear stresses. An arch might be subjected to snow loading at the crown and have a bending-moment distribution such as that shown in *Fig. 22*, so that over part of the arch the bending moment would have an approximately parabolic distribution. Corresponding to the bending moment there must be a shear stress. On a hypothetical strip, say, 12 inches long, in the direction of the arch, on a cross-section of the corrugation there would be a vertical snow load and radial components from the main arch stresses in compression, as the Authors had

shown; but, in addition, there would be the difference of shear across the strip, and that difference of shear would have a distribution approximately as shown in *Figs 23*. In the tension zone there would be cracks in the concrete. If the concrete were not cracked, there would be a parabolic distribution. The load was supported on the strip by the difference of shear in the direction of the arch across the strip, and that shear would accumulate from the section of maximum towards the section of zero bending and would act the same way as in a long cylindrical shell supported at the ends. In a long shell the shear was eventually taken up at the end by a diaphragm or edge beam. Should not consideration be given to those shear stresses in the design of large spans? Professor Baker was a little puzzled about it.

Some years previously, when dealing with a large hangar in Great

Figs 24



Britain, he had proposed an aluminium construction very similar to that described in the Paper. There were two main reasons for that. The first was that at the end of the war the aluminium factories had been looking for work and had been accustomed to curved construction; and the second was that, in aluminium, it was possible to construct very large spans, in corrugated arches, in just the same way as the London tubes were constructed—in segments, but each segment consisting of half the arch.

The joint was made as shown in *Figs 24*, and the half-arch was light enough to assemble on the ground. A travelling derrick could be used to erect the two sides of the arch. It seemed to Professor Baker that something of the sort should be tried in the building of a hangar or similar large-span roof structure.

Mr N. W. B. Clarke said that he had been interested by the development of the Ctesiphon arch type of structure and had been privileged to watch the construction of one example, of 60 feet span, and then to apply loading

tests to it. Somewhat to his surprise, the deflexions of the arch had proved to be very small under Code wind loadings applied, normal to the arch ring, inward on the windward side and outward on the leeward side. Unbalanced snow load had not been applied but, in view of the behaviour of the arch under wind loading, he did not anticipate that, with spans of 60 feet, there would be excessive deflexion under snow load in Great Britain, even if unbalanced.

The maximum test wind load had been twice the load corresponding to an 85-mile-per-hour wind. Yet the maximum deflexion had been less than ½ inch and the recovery more than 70 per cent. The movement of the arch was interesting; it took the generalized shape illustrated in *Fig. 25*, the maximum movement being somewhere in the region of X, where it would be expected with a uniform shell.

One or two points had occurred to him about future development. First, there was the ever-decreasing inclination of the thrust on the foundations as the arch became more shallow. With the very large arches which

Fig. 25

Mr Waller was contemplating that might raise difficulties unless he could ensure sufficient vertical load above the springings, or other means to depress the resultant thrust to a safe angle, or provide under-floor ties.

The next point concerned temperature movements. If the end walls were of brickwork, then as the temperature changed the crown of the arch would tend to move relatively to the top of the wall. That meant that there could not be a rigid joint between the gable wall and the roof. If there were a rigid joint, something would break.

Those considerations led to the idea of putting in a half-dome to fill up the end, which had the double advantage of reducing the wind loading and of ensuring that the movement of the end wall always corresponded to the movement of the arch itself. A half-dome might also be cheaper than a wall.

In the event of differential settlement occurring, how much twist could the Authors tolerate on any given section of the length of the building? The assumption was that the earth resistance was not uniform, and that as the foundations settled unevenly there would be a definite twist in the arch.

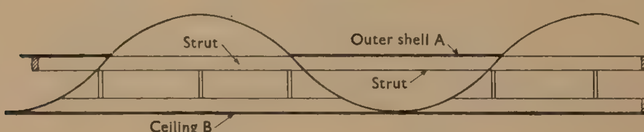
Mr Clarke said that his experience with sprayed mortar had not been

altogether satisfactory, because its quality tended to be uneven. Had there been sufficient experience of that technique in Great Britain to enable reliability to be placed on the durability and uniformity of such mortar in a complete structure ?

There had been a previous comment on drying winds and Mr Clarke confirmed that when the structure had been built at the Thatched Barn there had been one of those rare periods in England when there was a drying wind. It had certainly produced cracks in green concrete, and that was something to watch.

For many purposes a double shell would be an enormous advantage since a single very thin concrete shell could not offer much thermal resistance. It would be interesting to know, therefore, whether the Authors thought they could place a second shell at A in *Fig. 26*, possibly in conjunction with their suggested struts, together with a ceiling at B and so

Fig. 26



obtain a double skin right through. Those additions, if feasible, would also further stiffen the arch.

The Authors, in reply, said that in designing the type of construction described in the Paper the possible effects of both snow load and wind load—raised by Mr Blumfield—had to be considered.

A uniformly distributed snow load over the whole of the catenary arch produced no bending moments, and resulted in an increase of only a few lb. per square inch in the compressive stress in the concrete. The usual procedure had been to adopt an appropriate intensity of loadings over those portions of the arch where they would produce the maximum bending moments. A snow cap over the middle third of the arch produced the maximum positive bending moment at the crown, and snow drift against the lower slopes of the arch produced the reverse effect.

With regard to wind-load distribution, it was customary to follow the rules laid down in C.P. 3, Chapter V., because clients usually stipulated that. It was realized, however, that the type of structure under consideration was not of the conventional type for which the Code was primarily prepared, and the actual wind-pressure distribution was substantially different. But the stresses resulting from the actual wind pressures, as found by wind-tunnel tests, were no more severe, and were therefore well provided for in the design. The possibility of full internal pressure or suction inside buildings with large doorways was taken into account.

In addition to the direct stress which resulted from the bending

moments in the arch, there was a corresponding shear stress, as Professor Baker had explained, which had certainly been considered. It was regretted that, owing to limitations of space, the question of shear stress had been briefly disposed of in the Paper, but it was emphasized that that stress was everywhere of small magnitude. In a 300-foot span, for example, the maximum value of the shear stress was of the order of 20 lb. per square inch.

The distribution of shear stress had been calculated in accordance with the accepted principles, based upon an uncracked section, since tension rarely, if ever, occurred in the arch. The problem was similar to that of the long cylindrical shell supported at the ends, but with one important practical difference. In the corrugated arch, the shear stresses were produced by the superimposed loads alone, but in the case of the barrel-vault type of roof, the entire deadweight of the structure had to be transmitted to the supports by means of that shear stress; hence the severity of the stress in the latter case and the need for diaphragms.

Mr Blumfield had also asked why steel reinforcement had been used in smaller spans, and what was the largest span constructed without steel? In the early Ctesiphon-type arches, of 40 feet span, with shell thickness of only 1 inch, no steel whatever had been used, and those buildings were still in service. It was a little on the conservative side to use steel, but some reasons for the inclusion of a small quantity in subsequent buildings were given in the Paper. Up to the present, the largest unreinforced example was a 100-foot-wide building, 30 feet high, in two 50-foot spans.

With regard to costs, also mentioned by Mr Blumfield, the Authors said that details as to man-hours per unit floor area for various types of buildings, together with shell thicknesses commonly used, had been given in the Paper (see p. 160) and it was hoped that they might be a help when any particular building was to be estimated. It would be appreciated, however, that owing to the wide range of types of structure being built, under such diverse conditions and for such a variety of purposes, it was not only difficult but misleading to attempt a reply in definite figures. That, of course, applied to all the Author's work—after all, how much did it cost to move a yard of muck?

Experience showed that in many cases in spans up to 40 feet, those shells competed on very favourable terms, in first cost, with the lightest steel-framed asbestos-clad shed. In future cost—durability, fire risk, heating, repairs, and painting—the concrete shell manifestly had a distinct advantage. As spans increased the concrete shell definitely improved its position.

One example of interest that might be cited had just come to hand from India where offers had been made to the Central Building Research Institute at Roorkee for attractive small dwellings in quantity, at about 3*s.* 6*d.* per square foot of floor.

The cost of any system might be considered otherwise than in the

terms of a priced bill of quantities. All building costs were sensible to the following main considerations :—

1. Total weight of structure. It had been calculated, independently of the Authors, that a corrugated concrete shell house weighed between one-quarter and one-sixth of the weight of a conventional brick house.
2. The number of times the materials had to be handled. The in-situ arch, when the principal materials, sand and cement, were brought to the site, mixed, and laid straight in place, compared favourably in that respect.
3. The class of labour required. Wide experience had shown the almost complete adequacy of unskilled labour for that work. That was largely because of the simplicity of the falsework employed.

The last item might be illustrated by the following example :—

At Chikuni, said to be one of the most inaccessible places in Northern Rhodesia, the inmates of a leper colony had built themselves, from written instructions, not only houses but also a church, and were proceeding with a school 50 feet by 25 feet, roofed with a tied shell on block walls. Apologies were offered for the walls, which were essential for hanging up maps and diagrams.

What those buildings " cost " was hard to say : a little cement and gunny ; some local sand and bamboos for falsework ; and the voluntary services of the patients and a devoted padré. Such items did not amount to a formidable amount of cash. What they were worth to their proud constructors could be left to the imagination. The item for " profit " could not be stated in terms of money but—" all that glisters is not gold."

Mr Watson had mentioned the effects of end winds on a large hangar, and a possible solution to that problem involved the use of a separate framework.

An alternative solution preferred by the Authors in appropriate cases was to allow the arch to resist the pressure on the end walls, using the stiffeners shown in *Fig. 17* to distribute the reaction along the length of the building and prevent any concertina action of the end corrugations.

Referring to the remarks by Messrs Watson and Lewis-Dale about the application of those buildings to aircraft hangars, the Authors did not pretend to have any special knowledge of the lay-out of hangars, and valued the remarks that had been made regarding the difficulties involved. It was, however, surprising to find how adaptable the building could be, in one form or another.

The space at the sides of the hangar, where the headroom was insufficient for the storage of aircraft, could be used in many cases for workshops and offices. Where that accommodation would be insufficient, or where almost continuous side access was required, an ideal solution was

to be found in the use of the tied arch. The springings of the arch were raised up on to columns and lintels, and the outward thrust was resisted by ties or buttresses. By that means the full desired headroom was obtained over the whole area covered, and continuous access was provided along the sides between the columns, where buildings of conventional type could be added for workshops.

The tied arch was one solution to the problem of lifting tackle, for overhead runways could be attached to modified ties, or, where mobile cranes were used, allowed liberal headroom, which was of special interest when modern aircraft, with high tails, were considered. A further purpose served by the ties was in connexion with the support of the gable ends and main doors against wind pressures; by the addition of bracing between the first two ties, a lattice beam of great strength was provided at lintel height, lying in a horizontal plane, and capable of transmitting the end wind pressure to the sides of the building.

Mr Clarke had expressed concern that the decreasing inclination of the thrust on the foundations might raise difficulties as flatter arches were used. In the case of large-span hangars, it was not anticipated that the rise of the arch would be less than about one-fifth of the span, giving an inclination at the springings of about 40 degrees to the horizontal. When the weight of the foundations was taken into account, the resultant thrust to the ground was inclined at only about 25 degrees to the vertical.

With reference to climate control, Mr Hall's experience with his umbrella-cum-flat roof was very interesting; his experiment was fundamentally different from the proposal referred to in the Paper.

It would appear that the sun's rays beat on the corrugated iron and raised it to sun-ray temperature; the corrugated iron, being a good conductor, passed the heat on to the imprisoned air until, although in the shade, it became thoroughly "stewed"; as the sun went down, the air performed the function of a hot compress and effectively derived from the roof the beneficient effects of radiation and contact with the cool evening airs and "the position was no better than before!" It was agreed that if the purpose of the arch was to produce shade, it should be open-ended and oriented as suggested. In that case, corrugated iron would be practically as efficient as concrete, and so would a tree or a tent fly.

The construction proposed in the Paper was not to provide shade or to produce shade temperature. Its purpose was to form a close and well-insulated envelope—double shelled, or ceiled, or both—and thus to create the ideal conditions for air-conditioning. In such case, the cubic content was not the important factor; what counted was the total heat transference whether by conduction through the shell or by uncontrolled air-leakage.

In any given lay-out, that total heat transference was directly proportional to the exposed area of roofs and walls in the building(s) to be conditioned; the single large arch proposed had less exposed area than

that of the individual dwellings it sheltered and, in addition, the difficulties and losses arising from the wide-spread distribution of conditioned air, which could not be avoided with scattered dwellings, were reduced by simplification and concentration of the plan. Taking all into account, the Authors felt that, despite the additional amenities provided by the provision of conditioned air outside as well as inside the houses, their proposal would also prove cheaper than contemporary practice.

There was an opinion, frequently expressed and reflected in the discussion, that the arched form suffered from an inherent disadvantage in that it enclosed much unnecessary space; as a result, the conclusion was hastily reached that arched buildings presented peculiar difficulties in "space" heating, ventilation, and air conditioning. In the Authors' view, the opposite was the case. For example, in the case of a factory, store, transit shed, or bus depot, the choice of span available to the designer was almost unlimited; the shells could be single- or multi-span springing from ground level or raised on supports with thrusts taken by ties or buttresses and, which was of such immense importance, the rise of span ratio could be kept down to $\frac{1}{8}$ if desired, or less in the case of tied arches.

The closing date for correspondence on the foregoing Paper has now passed and no contributions other than those already received at the Institution can be accepted.—Sec. I.C.E.

WORKS CONSTRUCTION DIVISION MEETING

24 February, 1953

Mr David M. Watson, Member, Chairman of the Division, in the Chair

The following Paper was presented for discussion and, on the motion of the Chairman, the thanks of the Division were accorded to the Authors.

Works Construction Paper No. 23

"The Application of Precast Concrete to the Construction of Acton Lane 'B' Power Station"

by

John Anthony Derrington, B.Sc.(Eng.), A.M.I.C.E., and Arthur George Spicer Lance

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SYNOPSIS

The Paper describes how the framework of the turbine house and switch annexe was constructed of concrete units, precast on the site and erected by two travelling cranes. Concrete floor slabs and joints were then placed, tying the units together and making the structure act as though monolithic.

The design of the major units—turbine-house columns 66 feet long, weighing 33 tons, turbine-house roof beams spanning 65 feet and weighing 27 tons, roof slabs cast in sections each 26 feet long, 8 feet wide, and weighing 12 tons, and the other units in the multi-storey switch annexe—is described in detail, and the methods of providing in-situ concrete joints at all places is explained.

The two 35-ton cranes used for lifting the precast units are described, and details given of the way in which the precast members were lifted into position. Details of the various attachments devised for lifting and guying the concrete units in position are also given.

One section deals with the concrete mixes and the concreting plant, and test results obtained. The speed with which the job was carried out is shown, and the advantages of this type of construction and its effect on the site methods are discussed.

INTRODUCTION

THE practice of precasting concrete members, either on the site or in a factory, and erecting them to form a structure is by no means a new process, although it has received great attention in the past few years.

Nor is the construction of a power station turbine house in reinforced concrete a new departure, for the British Electricity Authority and its predecessors have several examples to show where reinforced concrete was used as the material of construction.

At Hams Hall "B," near Birmingham—one of the largest single stations in Great Britain—the entire structure, with the exception of the chimney

shaft, was of reinforced concrete, and at Drakelow, near Burton-on-Trent, and Skelton Grange, near Leeds, reinforced concrete was used in the turbine house construction—at the latter in the form of a shell roof.

Although precast concrete units of larger size have been used in jobs before, the Authors have no knowledge of a structure in which concrete members of the size used at Acton Lane have been precast and erected by derrick cranes in a manner similar to that used for structural steelwork and at a comparable speed. When it is remembered that the original structural steelwork design was changed to reinforced concrete from considerations of speed and economy, this structure is quite remarkable.

Description of New Station

Acton Lane Power Station, first commissioned more than 50 years ago, is situated in north-west London on the Grand Union Canal, and has an installed plant capacity of 158 megawatts. The new "B" Generating Station will, in stages, eventually replace the existing plant, some of which is nearly 30 years old, and will consist of six 30-megawatt, 11·6-kilovolt turbo-alternators, and nine 240,000-lb.-per-hour stoker-fired boilers. The latter will provide steam at 600 lb. per square inch and 850° F., giving an expected overall thermal efficiency of 25½ per cent.

The first turbo-alternator will be commissioned early in 1954, and the remainder in stages. The completed station will have three new reinforced-concrete cooling towers, each with a capacity of 2¼ million gallons per hour, and an existing tower of 1½-million-gallons-per-hour capacity.

The first half of the building now under construction will accommodate three of the 30-megawatt alternators and five boilers, and can be treated as three structural units—the boiler house, the turbine house, and the switch annexe. The frameworks of the last two, which have been designed and constructed in precast reinforced concrete, form the subject of this Paper.

A steel framework is used for the boiler house, the coal bunkers, and the substructure of three 300-foot-high reinforced-concrete chimneys.

Since the present structure is only one-half of the complete building, there is a temporary end at the limit of the first section with facilities for extension later.

A separate building for a stores-and-workshops block, constructed as a two-bay portal frame in precast concrete, was also built on the same site.

Adoption of Reinforced Concrete Construction

The original design for the framework of the building called for structural steelwork, and the foundations and retaining walls were designed on this basis. The ground consisted of a soft brown clay, 27 feet thick, overlying a stiff blue clay of considerable thickness.

The main operating floor level of the turbine house was 120·00 O.D.,

approximately 3 feet above original ground level, and the main basement level was 23 feet below. The foundations were taken down into the blue clay using a bearing pressure of $2\frac{3}{4}$ tons per square foot. A cantilever reinforced-concrete retaining wall, 32 feet high and 6 feet thick, was formed around the perimeter of the building. Columns inside the building were generally carried on strip foundations the thickness of which ranged from 8 feet to 13 feet, and the 18-inch basement floor between them was placed directly on a mass-concrete sealing-coat.

The main foundations contract was placed, and foundations and retaining wall were built by conventional methods, the latter being constructed in a girdle trench 20 feet wide; afterwards the dumpling was excavated and the main foundations concreted. Pockets were formed for the holding-down bolts required for the steel stanchions, and reinforced-concrete trenches were formed in the basement floor to take 8-foot-6-inch-diameter cooling-water pipes.

Tenders were then invited for the supply, delivery, and erection of the structural steel frame. At that time it was necessary to make great economies in the use of steel and, owing to the demands made upon the industry, long delays were occurring in the delivery of fabricated steelwork.

Permission was obtained to submit a scheme in prefabricated reinforced concrete for the turbine-house and switch-annexe frames, the erection of which would be planned to avoid interference with the steel erection progress in the boiler house.

The scheme was prepared on the principle of precasting all units on the site and erecting them with derrick cranes, since the contractors favoured this method for speed and economy. The amount of scaffolding required would be reduced to the minimum, and the cost and delay of fabricating a steelwork travelling platform for supporting the turbine-house roof shuttering would be saved. This proposal, when prepared, was checked closely by the British Electricity Authority to see that it met all requirements; since it showed a saving in money, offered a contract period of 36 weeks, and reduced the amount of steel required by two-thirds, the order was placed for its execution.

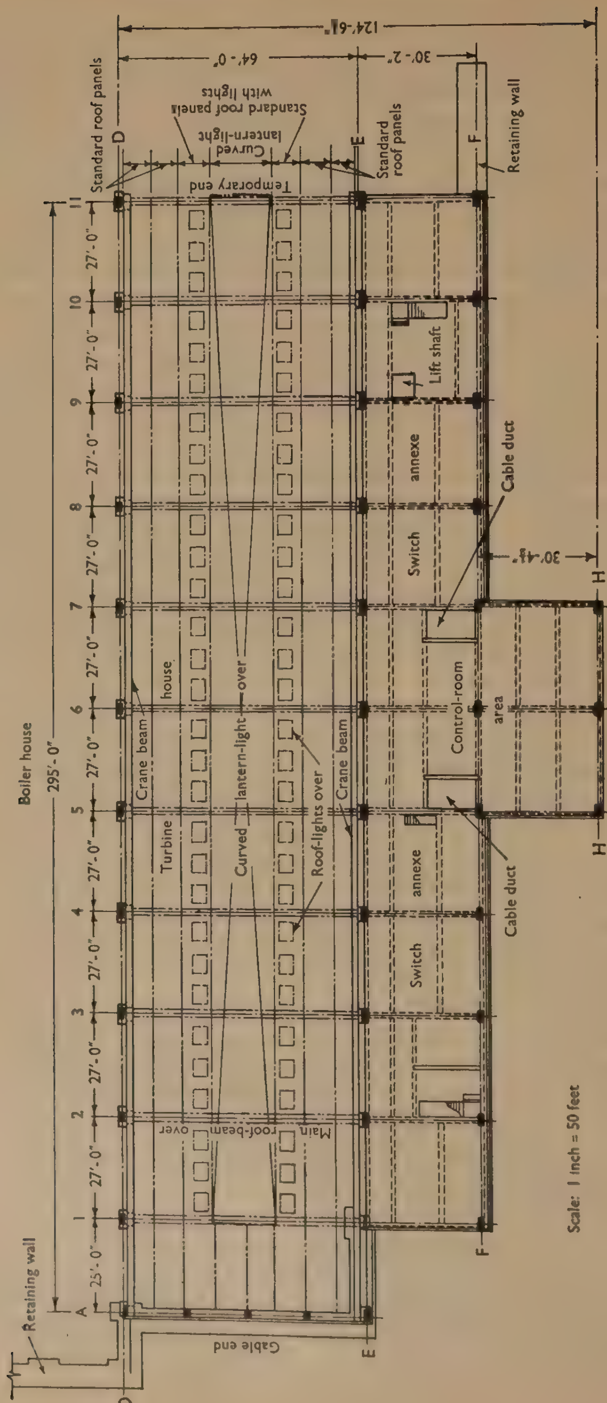
Since originally a steel framework had been envisaged, alterations to the foundations were required and allowance for these was made in the tender submitted. The overturning moments which the columns would impose on their foundations had also to be resisted, because, when the precast columns were set in position and concreted into the pockets in the foundations, their only stability, until the framework was completed, was the fixity provided at the base.

Design of Precast Members and Joints

General Description

Figs 1 and *2* give a plan and typical section through the turbine house and switch annexe. It will be seen that the turbine house consists of two

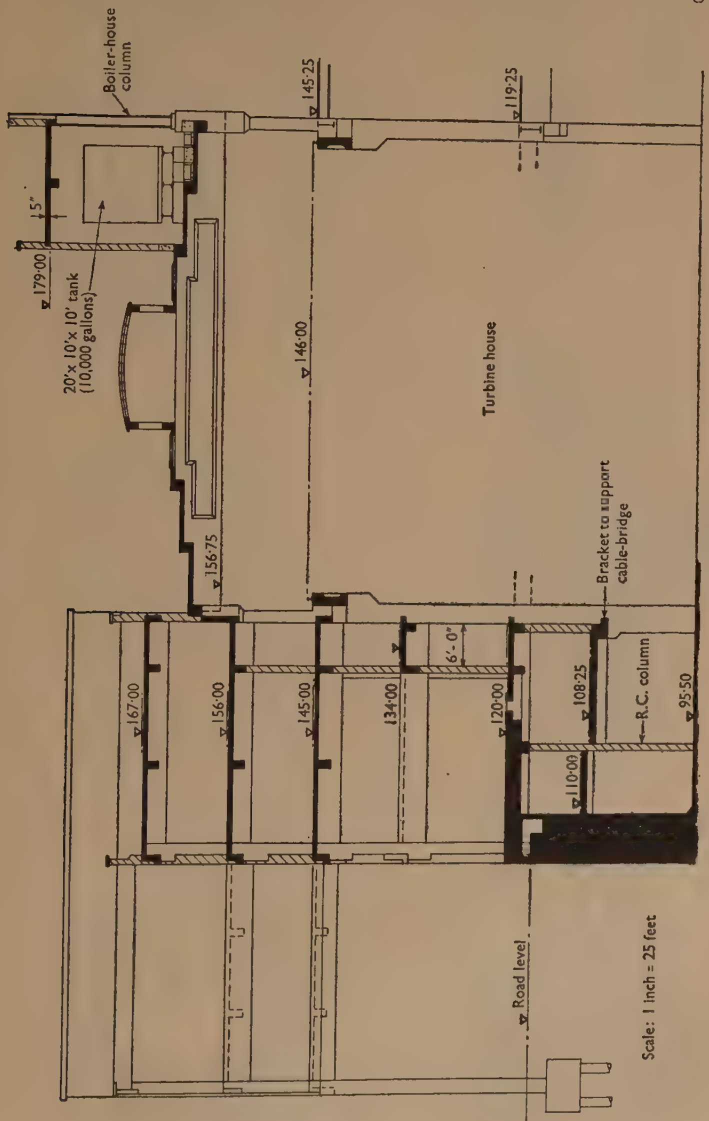
Fig. 1



Plan of Turbine House and Switch Annexe

rows of principal columns at 64-foot centres, spaced longitudinally in 27-foot bays. The columns in the south row (D-line) carry 360 tons each, transmitted from the boiler-house section, in addition to the loading from

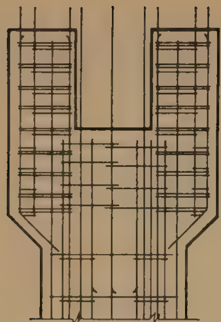
Fig. 2



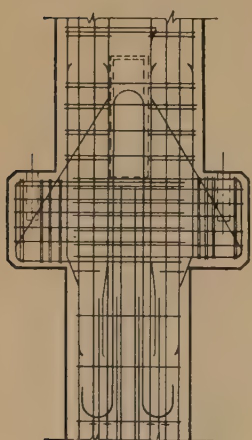
CROSS-SECTION THROUGH TURBINE HOUSE AND SWITCH ANNEXE SHOWING POSITION OF SURGE TANKS

the turbine-house roof and crane. The roof is carried in a single span across the building by precast beams, whose soffits are 61 feet 3 inches above basement level. An electrically operated travelling crane, capable of

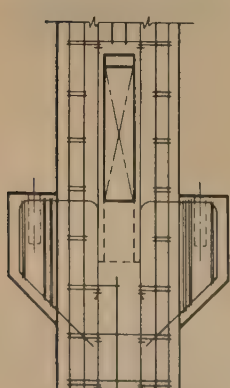
Figs 3 (a)



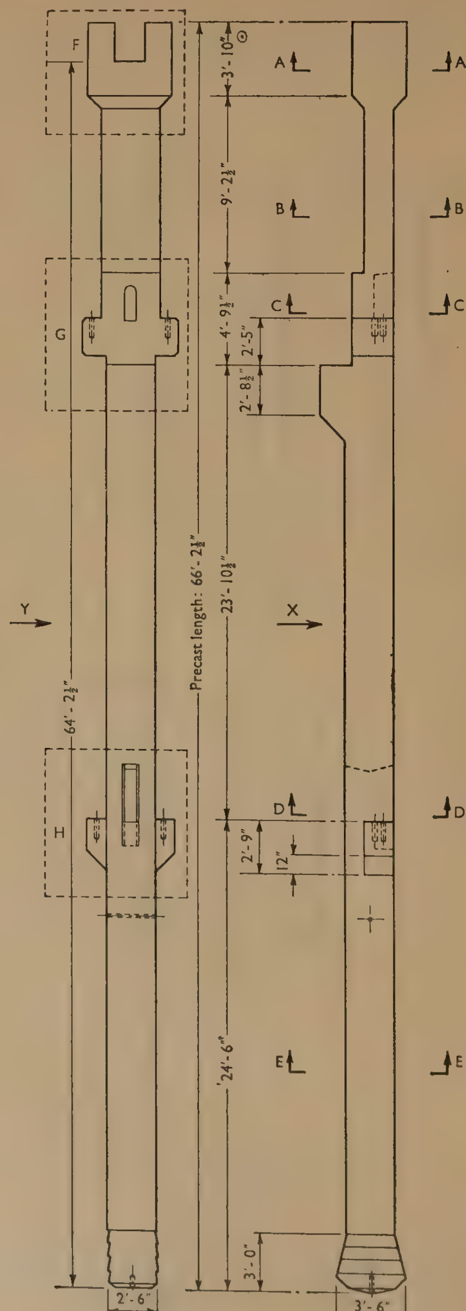
DETAILS OF REINFORCEMENT
AT F



DETAILS OF REINFORCEMENT
AT G



DETAILS OF REINFORCEMENT
AT H
Scale: ⅛ inch = 1 foot

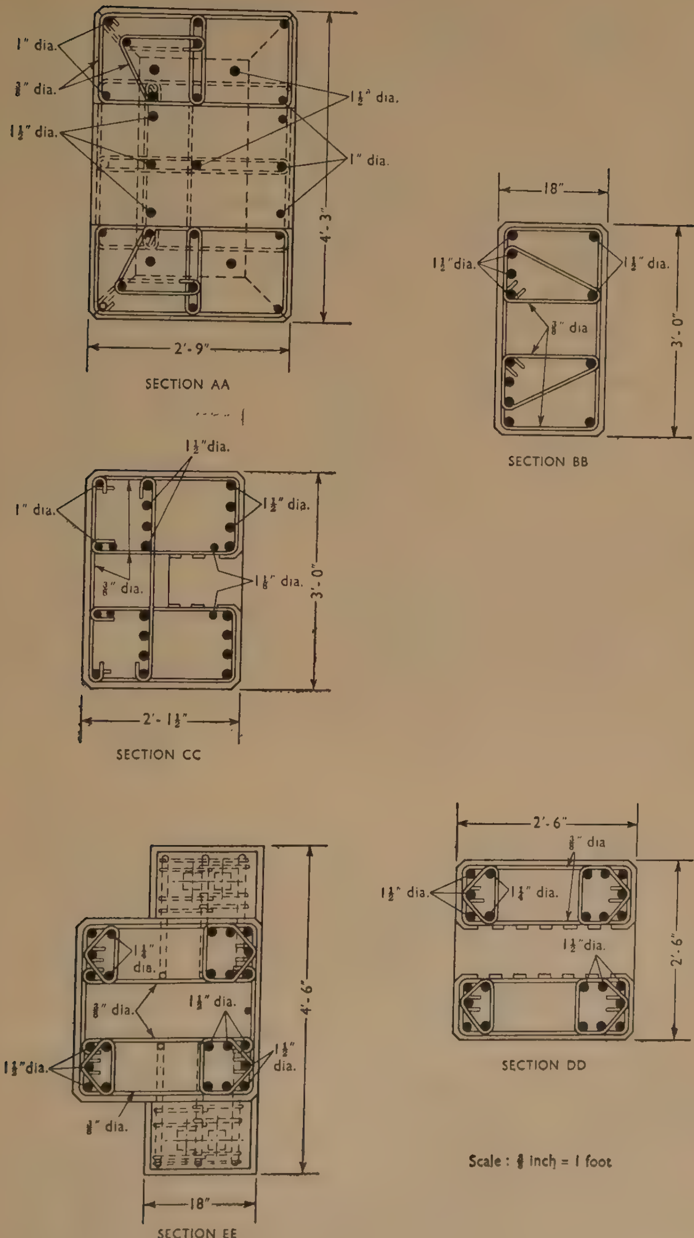


ELEVATION X ELEVATION Y

Scale: 1 inch = 10 feet

DETAILS OF PRINCIPAL

Figs 3 (b)



Column Row D

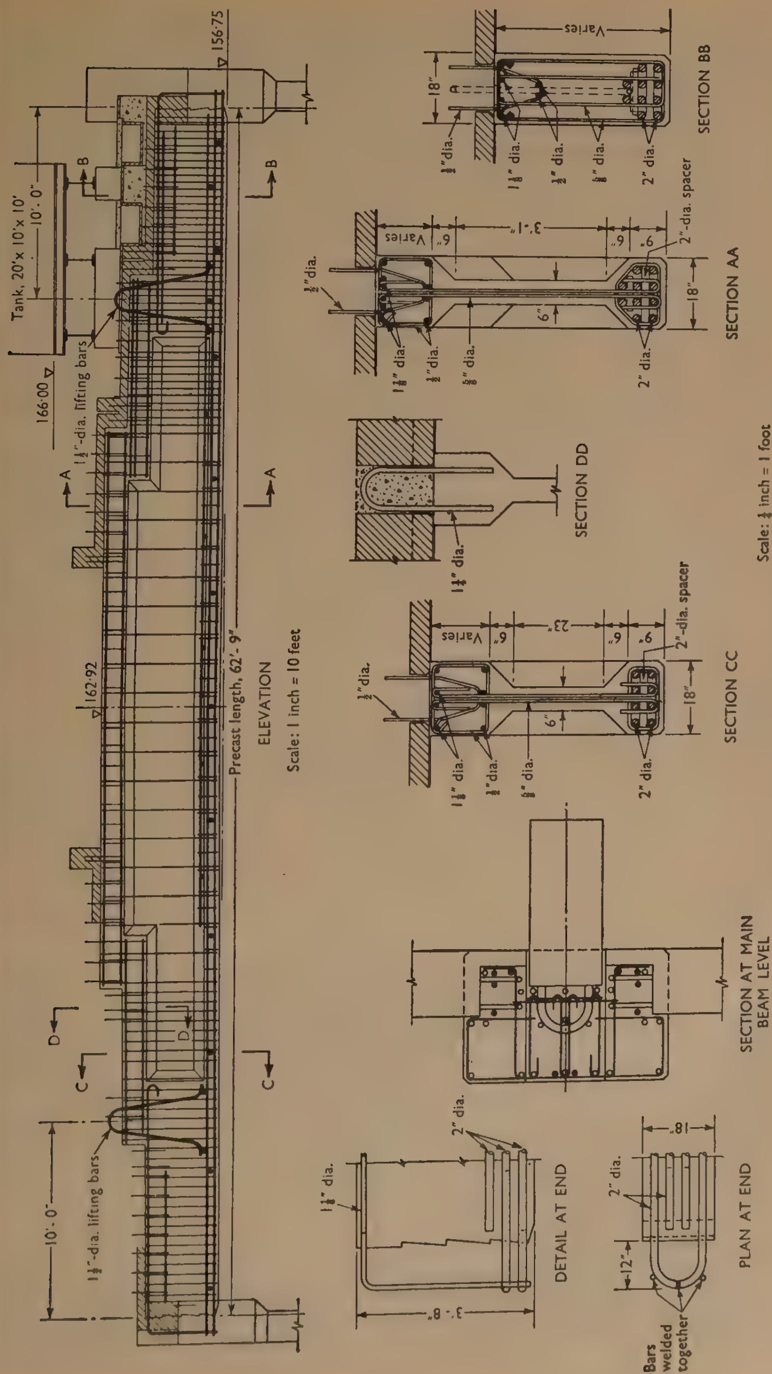
lifting 70 tons (the heaviest part of an alternator), travels on precast concrete crane-beams at 59-foot-6-inch centres, which are carried on brackets formed on the main columns, 46 feet above the basement level. The operating floor is 23 feet above basement level, and is supported on steel joists. On the other side of the turbine house (E-line) the columns support the switch annexe. It is 30 feet wide, and is also carried by a further row of columns (F-line) at corresponding centres of 27 feet; generally, it is a three-storey building with an additional floor over part of its length. The switch annexe consists of ten bays, each of 27 feet, and the turbine house has a further 25-foot bay at the east (gable) end.

All columns and main and secondary beams were designed and cast as single units, and were precast in timber moulds on the ground. The members were erected and trued into position; then the concrete joints and floor slabs were cast in situ. Reinforcing bars left protruding from precast members were lapped at the joints, thus making beams, columns, and floors act as a monolithic structure. The columns were temporarily guyed in position until concreted solid into their bases, and the framework was designed so that each horizontal member could be fitted into a slot or rested on a bracket without any props or other temporary support. The frame was capable of remaining stable until the in-situ floors and joints completed the structure, after which it was ready to receive the brickwork cladding, and the loading from machinery, etc. Measurements taken during a strong wind before jointing the structure revealed negligible movement. The wind forces on the completed building and transverse loads from the crane surge are taken by 9-inch-thick brick walls the full height of the switch annexe at alternate column positions. The precast and in-situ slabs, acting as horizontal girders, distribute these loads equally throughout the structure.

Turbine House Columns

Figs 3 show typical details of a main D-line column. From foundation to crane level it was 30 inches by 30 inches, and from there to roof level it was 36 inches by 18 inches. The total precast length was 66 feet and the weight about 33 tons. Holes and brackets were provided to receive steel joists from the boiler-house structure. Two of these brackets or "ears," situated immediately above the crane-rail seating one on each side of the column, were used as lifting points.

The top of the column was bifurcated to receive the main roof-beam with reinforcement protruding into the jointing space. In addition, further reinforcement projected for extending the column in situ to provide the seating for the boiler-house column. To meet the varying conditions of live and dead loading and to achieve the greatest economy in use of steel, the reinforcing bars were placed asymmetrically in the section. The total reaction from the crane beams was 112 tons with considerable eccentricity, and the reinforcement was disposed so that under maximum loading the



DETAILS OF TURBINE HOUSE ROOF BEAM

resultant force coincided with the centre of gravity of the section. The highest stresses caused by eccentric loading did not, therefore, coincide with the most severe direct stress. The reinforcement in the unloaded face of the column was in some cases controlled by the amount required to resist the bending forces, either during lifting or during the early stage of construction before the brick diaphragm walls had been built. Thus a full analysis of each individual case was necessary; this saved more than 30 per cent of the steel which would have been required by the simple design with equal reinforcement in both faces of the column. The maximum load on a column was 764 tons, requiring main reinforcement of twenty-six $1\frac{1}{2}$-inch-diameter bars equalling 5·1 per cent of the concrete section.

The slots in the column for steel beams will be filled with a dry-mix fine concrete, well hammered-in. For purposes of design, however, the slots were deducted from the full concrete section and additional bars inserted at these points to take the higher unit stress.

The stress on the column section of 120 tons per square foot was reduced to 30 tons per square foot, which the foundation concrete could take by bulbing, notching, and splaying the foot of the column. The size of the pocket cut in the foundation and the proportions of the serrations on the column foot were decided by these requirements. The shape of the foot allowed the concrete filling to penetrate right underneath without entrapping air. The filling was a fine concrete made with $\frac{3}{8}$-inch-maximum aggregate which contained an admixture to reduce surface tension. This concrete was vibrated with a pencil-type vibrator.

The E-line columns, which carry several heavily loaded floors in the switch annexe, are similar in size. The main floor-beams are deep and floor-to-floor heights are low. Each column had a continuous vertical channel formed in the switch-annexe face, and at beam-soffit levels concrete ledges were constructed across the channel to provide seating for the beams. The sides of the channel were keyed and had light reinforcement projecting to secure the concrete infilling, which was placed in short lifts after the beams were set.

This form of construction received special attention because of the effects of shrinkage in the infilling, but the conclusion was reached that it could be considered as a monolithic section in resisting the final condition of loading. Since there were no signs of hair-cracks between the precast and in-situ concrete on completion, this conclusion appeared justified.

Where the columns on this line required extension above the turbine-house roof level, they were raised with in-situ concrete in a manner similar to the seatings for the boiler-house columns.

Turbine House Roof Beams

The main roof-beams of the turbine house shown in *Figs 4* were shaped to fit the terraced profile of the roof. They were designed as free-

ended to carry a superload of 30 lb. per square foot of roof area supported. Mild-steel reinforcing bars of 2 inches diameter were used in tension and the whole of the shear carried on $\frac{5}{8}$-inch-diameter vertical stirrups. The precast roof-slabs could not act as a compression flange, since the majority of the load was placed on the beam before the joints between beam and slab had been made. Compression reinforcement was, therefore, required in the top flange. Because of the varying section, uniform reinforcement was needed throughout the length of the beam. The dead weight was, however, reduced by this means.

Since 2-inch-diameter bars longer than 65 foot 3 inches were not available in time, the ends of the main bars were looped and welded together to form semicircular hoops, developing adequate bond and forming a positive mechanical lock with the bars projecting from the tops of the columns. Additional lacing reinforcement was provided before the in-situ concrete was placed in the joint. Notchings were introduced at the end faces to obtain the area required for bearing.

Each beam was 63 feet long, 6 feet deep at the centre, and weighed 27 tons. Most of these beams were designed to carry 10,000-gallon surge-tanks which were sited on the turbine-house roof in penthouses with 9-inch brick walls and 6-inch concrete roofs. For economy in shuttering, all roof beams had the same outline and the reinforcement was varied to suit the loading.

Two bars, $1\frac{1}{4}$-inch diameter and 10 feet long, were introduced, lapping with column bars and carried over into the joint on top of the beam to provide a nominal amount of continuity over the column head. This was to stiffen the structure until the brick diaphragm-walls were built. No reduction was made in the mid-span moments of the main beams.

Turbine House Roof Units

The turbine-house roof consists of precast elements, 26 feet long and 8 feet wide, in which the slab and secondary beams are incorporated. The slab spans 8 feet between the secondary beams which in turn span on to the main roof-beams. Each unit has one upstand edge beam and one downstand to form the terraced roof. A minimum solid-slab thickness of 5 inches was specified for these units and they were designed as free-ended to carry a superload of 50 lb. per square foot. They were cast so that when set in position there would be sufficient cross-fall on the roof, and were made waterproof by a $\frac{3}{4}$-inch-thick covering of asphalt.

Small-diameter reinforcing bars were left protruding from the ends of the units to lap with bars projecting from the tops of the main roof-beams. After erection these bars were bent down to interlace, and the gap between units was filled with fine concrete, giving lateral rigidity to the main roof-beams and making the roof slab act as a horizontal girder. This

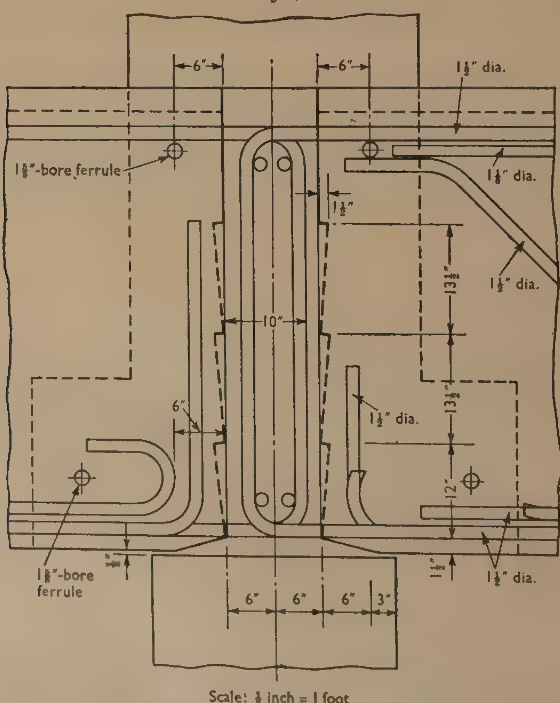
advantage would have been lost if the more common hollow precast concrete beam had been used instead.

Larger bars were looped out of the ends of the stiffening ribs and meshed with others from the main beam. The 2-inch-wide joints between units were filled with fine concrete and reinforcing bars were inserted over main roof-beams to provide nominal continuity. A central curved lantern-light was cast in situ, and the roof units on each side of this were made 6 inches thick to take a glazed pavement-light of similar pattern.

Crane Beams

The beams supporting the overhead crane were cast 26 feet 6 inches long, 4 feet 6 inches deep, and 18 inches wide, with a 6-inch-deep panel inset on the turbine-house face. The maximum wheel loads used in the design included an additional 25 per cent of the total load for impact. Cross-surge was taken as 10 per cent of the combined crab-and-load weight, and was divided equally between the wheels. The beams were designed as free-ended and the main reinforcement was ten $1\frac{1}{2}$-inch-diameter mild-steel bars in tension, and six $1\frac{1}{8}$-inch-diameter bars in the top flange

Fig. 5



CRANE-BEAM CONNEXION AT COLUMN BRACKET

Fig. 7

Inside of Turbine House after Construction of Lantern Lights and Roof Lights

Fig. 9

General View of Precasting Yard

Fig. 11

General View of Site 10 weeks after Erection commenced

Fig. 14

Workshops and Stores. Frame complete. In-situ Floor Slabs and Beams being concreted

Fig. 15

General View of Site looking East. Erection commencing 23 October, 1951

Fig. 16

View of Basement. 26 October, 1951

Fig. 17

26 November, 1951. Five Weeks after Erection commenced

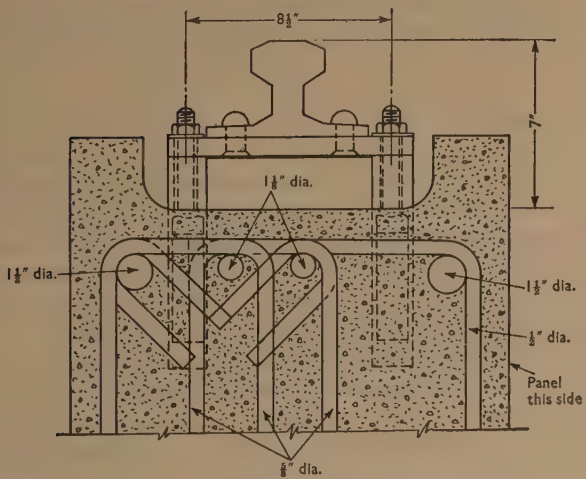
Fig. 18

17 January, 1952. Twelve Weeks after Erection commenced

for compression and side bending due to lateral forces. The shear force was high and was taken entirely on $\frac{5}{8}$-inch vertical stirrups.

Fig. 5 illustrates the method adopted to develop the main bars in bond at the ends of the beams. It can be seen that two bars were looped out of the bottom flange up into the top of the beam, and these bars also provided a positive interlock with large bars protruding from the column bracket. The ends of the crane beams were notched as in the main roof-beams to spread the reactions over the column bracket. The filling to the joint was carried out with a dry concrete mixed with $\frac{3}{8}$-inch-maximum aggregate which was well vibrated.

Fig. 6



DETAIL OF FIXING CRANE RAIL TO CRANE BEAM

The fixing of the crane rails to the concrete beam provided a further problem. 112-lb.-per-yard flat-bottom rails were riveted to a $\frac{3}{4}$-inch-thick steel bearing-plate. This was secured to the concrete beams with $\frac{3}{4}$-inch-diameter expanding bolts, and since the holes for these were drilled to templet afterwards, great care was necessary in positioning the reinforcing bars, so that they would not interfere with the spacing of the bolts. A continuous groove was left in the top of the beam and this was packed with fine concrete, when the rail had been set. *Fig. 6* is a detail of the top of the beam and shows this fixing.

Roof Lantern and Lights

The cross-section of the building shows the curved roof-light, 270 feet long and 16 feet wide, which runs down the centre of the turbine-house

(*Fig. 7*), concrete columns rising from the main beams and concrete longitudinal beams connecting them, were cast in situ to provide the support. The curved lantern was constructed with 12-inch-square and 1-inch-thick pressed annealed lenses cast within reinforced-granolithic-concrete ribs at 15-inch centres giving a net daylight area of 56 per cent. It was designed to carry a superload of 30 lb. per square foot. The light was sub-divided transversely by corrugated-copper expansion joints to suit the area of work which could be done in one day. The precast roof units on each side were provided with three rebated holes, 4 feet by 5 feet 3½ inches, to receive glazed pavement-lights which were constructed by the specialist subcontractor who designed and built the lantern. The flat lights were designed for the same superload (50 lb. per square foot) as the typical roof units. The bay at the gable end was not provided with roof lights and the 4½-inch-thick concrete wall to the lantern-end was constructed in situ on the top of the main roof-beam after erection.

The Switch Annexe

Along the north side of the turbine house is the switch annexe, 270 feet long and 30 feet wide. The precast concrete frame was erected at the same time as the main building and includes columns cast in one piece from ground level to a height of nearly 60 feet. There are four suspended floors except in the central control room, 60 feet wide by 54 feet long, and over the three bays immediately west of it, where there is an additional storey. The outside is clad with 13½-inch brick walls, and there are also 9-inch brick cross-walls at alternate columns.

Columns, main beams spanning 30 feet, and secondary beams spanning 27 feet were precast, and the solid floor slabs and staircases were cast in situ to tie the structure together. Floor loadings are heavy, ranging from 112 lb. to 280 lb. per square foot, and transformers weighing 75 tons each are also supported.

The main columns are at the same spacing as their counterparts in the turbine house, and were generally of 36-inch-by-15-inch section, ranging in height from 40 feet to 60 feet according to the number of floors. They were designed with reinforcement placed asymmetrically on the same basis as the turbine-house columns. Slots at each floor level provided a bearing for main beams, and brackets on the outside face carried the secondary beams supporting the external brickwork. These slots were filled with concrete during the casting of the floor slabs. Reinforcing bars were left projecting from the slots at each floor level for lapping into the main floor-slabs. At the points where the walls on each side of the control room returned, beams from four directions were supported, and there was some difficulty in ensuring that reinforcing bars did not clash and that the erection was straightforward.

The F-line columns were set in pockets cut in the top of the retaining wall, which was later raised to ground-floor level, thus concreting them in

position. The columns weighed up to 21 tons each and because of their slenderness, the reinforcement was in some cases dictated by the stresses during erection.

For the three columns supporting the outer face of the Control Room, separate foundations carried on bored piles were provided.

Main beams were large owing to the heavy loads carried, the typical section below slab level being 36 inches by 14 inches, and the main reinforcement consisting of twelve bars, $1\frac{1}{2}$ inch diameter. The beams were precast with main bars projecting at the ends to develop bond, and stirrups projecting from the top to tie into the floor-slab reinforcement. They were cast in $1{:}1\frac{1}{2}{:}3$-mix concrete and were designed as simply supported T-beams working in conjunction with the in-situ slab which was cast later. This basis of design is justified by recent tests on similar composite sections. To avoid overstressing the main steel, the beams were propped in position so that no deflexion took place until the slab had matured.

Ends of the beams were notched as in the turbine house, and rebates were also provided in the face to provide a seating for secondary beams. In all cases, bars were inserted through the joints into the slab to produce nominal continuity at the beam ends.

Secondary beams spanning 27 feet were precast up to the soffit of the slab, with stirrups projecting from the top as in the main beams. Size varied according to loading, typical sections being 21 inches by 12 inches with five bars, $1\frac{1}{8}$ inch diameter, as reinforcement. They were set in rebates left in the main beams and grouted in before the floor slab was cast. Since it was impossible here to carry part of the main tension steel through the support, top bars were inserted in the slab over the main beams to introduce partial continuity. The secondary beams were designed as T-beams, full advantage being taken of the flange provided by the slab concrete.

Floor slabs generally were 6 inches thick and were of 1 : 2 : 4-mix concrete, since the stresses were not excessive. They were cast on steel shutters supported on the secondary beams which also were propped to prevent deflexion until the concrete had matured.

The sub-basement main beams were cast in place with the slab because there was some delay in obtaining the design requirements of the electrical contractors. For the same reason the panels of the ground-floor slab carrying the transformers were omitted. This meant that the main beams in this floor had to be designed for the interim condition (that is, partly loaded as a rectangular beam of reduced depth) and in some cases compression steel in the top of the beam was necessary.

The concrete staircases were originally planned as precast units, to show a substantial saving. However, owing to lack of space in the casting yard, and because stair flights could not be precast when the casting of the floor slabs was delayed, they were cast in situ in the same sequence as the floors.

The control room, which is in the centre of the switch annexe, needed a clear space without columns, 54 feet by 60 feet, with 6 feet headroom for cable runs and tanks above the ceiling slab. It was, therefore, decided to use structural-steel lattice-girders for supporting the double roof. The steelwork was set on holding-down bolts left in the columns and roof beams. Approximately 30 tons of fabricated steelwork were needed and it was independently erected some weeks after the main structure had been completed.

The design of the whole frame was based on British Standard Code of Practice C.P.114 (1948).

CONCRETE

Concrete mixes used and works cube-strengths specified are given in Table I.

TABLE 1

Location	Nominal mix	Min. works cube-strength at 28 days: lb. per sq. in.	Permissible stresses	
			Compression, direct	Compression, due to bending
Turbine House Precast columns, Precast main roof-beams	1:1:2	4,500	1,140	1,500
Turbine House Precast roof-slabs *Switch Annexe* Precast beams	1:1½:3	3,750	950	1,250
Switch Annexe In-situ floor slabs	1:2:4	3,000	760	1,000

All the above mixes and strengths are those specified in British Standard Code of Practice C.P. 114 (1948) for aggregates complying with B.S. No. 882.

The aggregates used were natural flint gravels and sands from the Thames Valley. The nominal aggregate size for all mixes was ¾ inch maximum with the exception of the in-situ beam and column joints, where a "fine" concrete with ⅜-inch-maximum aggregate was specified.

The cement was ordinary Portland cement and was found in all cases to comply with B.S. No. 12. Laboratory tests were made on concrete samples and these showed that with the 1:2:4 mix the use of normal-hardening cement would be advisable, but with the 1:1:2 mix the use of a slower-hardening cement would give adequate strength.

Concrete materials were batched by weight, the nominal volume proportions being converted with appropriate allowances for moisture in the aggregate. The weigh-batching plant had a storage capacity of 23 tons each of fine and coarse aggregates, and 8 tons of cement. Sand and gravel were fed into hoppers by means of a 5-ton steam crane fitted with a grab, and the cement, which was supplied in bulk in 3-ton containers, was lifted on to the top of a cement silo by the same means. The three storage hoppers were in line, with cement between the two aggregates. A hand-propelled weighing and collecting trolley, fitted with a 2-ton dial-gauge, was mounted on a runway beneath. The mixer was a 28 N.T. machine equipped with a rising hopper into which the trolley discharged.

The concrete was delivered from the mixer drum into an agitator and through this into the cone of a 6-inch concrete-pump, which distributed the concrete to the casting beds in the basement of the turbine house. Although the concreting plant could mix and distribute concrete at a high rate, the production was limited by the number of precast units available for casting.

The 1:1½:3- and 1:2:4-mix concretes had a slump of about 3 inches. The 1:1:2 concrete had a greater slump in order to ensure that the best possible surface finish would be obtained. Since the lower flanges of the main beams were heavily reinforced, a very workable mix was necessary, although both internal and external vibrators were used. Both types of vibrators were used on all precast members and advantage was taken in the precast columns of the clause in C.P. 114 which allows an increase of 10 per cent in the concrete stresses where vibration is used for compacting the concrete. No cracking or crazing of the concrete has occurred.

Table 2 gives a summary of the test results obtained with cubes made on the site from samples of the concrete used.

TABLE 2

Nominal mix	1:1:2	1:1½:3	1:2:4
Proportions by dry weight	1:3·25	1:4·88	1:6·50
Average of 28-day cube strength	5,560	5,200	3,920
Number of tests	48	38	16
Coefficient of variation	9·8%	14·1%	Too few results

The high coefficient of variation in the 1:1½:3 mix was caused by the use of normal-hardening cement in in-situ work, and of slower-hardening cement in precast work.

Casting Yard Lay-out

In order to save the cost of transport and the delays in double-handling the precast units, the casting was planned so that the derricks could pick them up from the position in which they were cast and set them directly in their final place in the structure. Figs 8, Plate 1, include a plan of the job showing the positions in which the units were cast, and gives some idea of the congested nature of the basement and the difficulty of arranging the units so that the above objects were achieved. It is found that on all jobs of this type the most difficult site problem to be solved is this one of planning the casting yard in relation to the erection procedure and programme. *Fig. 9* is a general view of the precasting yard and shows the main roof beams, main columns, and roof slabs awaiting erection. The bridging of the shutters over the cooling water trench can be seen. At the right hand side the shutter for the E-line columns is ready for concreting.

The heaviest lifts were the main columns which weighed more than 33 tons each, and their position in the casting beds was dictated by the erection procedure described later.

It will be seen that the columns as actually cast were not at the same inclination to the axis of the building as originally planned. This fanning out of the casting positions was a development that occurred on the site owing to the small space available, and permitted the casting of the last columns at the west end of the turbine house, where the battered side of the earth face left at the temporary end of the building limited the space available.

The main roof-beams weighed 27 tons, but since these were to be lifted by the two cranes working together, the load was not critical. They were cast obliquely across the building.

The crane beams were cast in a position suitable for lifting by the derricks after they had erected the main roof-beam and columns and before they travelled to the next bay. In some cases it was found necessary to double-handle these units.

The main beams and secondary beams for the switch annexe and the columns on F-line, were all cast on a concrete mat at normal ground level at the top of the retaining wall, in such a position that they could be picked up by the derrick as it made its progress along F-line.

Figs 8, Plate 1, show that a cooling-water trench, about 15 feet wide and 12 feet deep, ran longitudinally through the turbine house, and this complicated the casting of the units, since all shutters crossing it had to be carried over it on tubular steel scaffolding and timbers. The delivery pipe from the concrete-pump had also to be taken down to basement level and across the trench on a tubular scaffold support. To achieve a true surface for the casting beds, the concrete sealing-coat in the basement was given a thin screed.

The shutters were all of timber, and the high quality of finish to the

concrete face was obtained with a wall-board lining, since plywood was not then available. It was found necessary to renew the lining after only six uses in order to maintain the required standard of finish. The columns were all cast so that the top trowelled surface did not face the turbine bay after erection.

Erection Procedure and Lifting Tackle

The contractors carried out the erection with two 35-ton steam derricks set on gabbards in order to give the necessary height.

The design for the reinforced-concrete framework proceeded accordingly, and it was found that the weight of the precast units could be kept within the 35-ton limit. In all cases it was necessary to make calculations for the weight of the member with allowance for the amount of reinforcement, since in some members up to 18 per cent of their total weight was accounted for by the reinforcing bars.

Figs 10, Plate 2, give details of the cranes which had the following lifting powers :—

(1) With single rope over top pulley.
Capacity : 10 tons at 50 feet radius.
(2) With single rope over centre pulley.
Capacity : $12\frac{1}{2}$ tons at 60 feet radius.
(3) Hoist rope over centre pulley and return block.
Capacity : 20 tons at 30 feet radius.
(4) Hoist rope over lowest pulley and return block.
Capacity : 35 tons at 24 feet radius.

In all cases the jib rope was taken to the lowest pulley and the return-block rope to a pin fixed 3 or 4 feet below this level.

The crane on the switch-annexe side was on concrete gabbards and travelled along the top of the retaining wall, but to give the required reach, the crane on the boiler-house side, being at a lower level, was mounted on steel gabbards 30 feet high.

The use of steam-driven derricks was decided upon in preference to diesel-driven or electrically driven machines, owing to the greater smoothness with which they could be operated. It was found that in practice, they could be "inched" with great accuracy and the fear of hoisting operations being interfered with by an electrical breakdown was also removed.

A mobile crane fitted with an 80-foot jib was used for placing the secondary beams in the switch-annexe frame and this enabled the derrick on F-line to keep pace with its partner on D-line. Afterwards, for the erection of all units in the workshops-and-stores building, the same mobile crane was used.

The main columns were cast in a position where they could be erected without double-handling. At approximately two thirds of their height,

concrete brackets had been formed on the sides to receive the steelwork from the tank-room floor, and advantage was taken of these for the erection. Steel yokes, of welded construction built-up from rolled sections and mild-steel plates, were designed to fit around the brackets, and all joints were pinned to facilitate fixing and removal. The main weight of the column was taken up to a short spreader beam made of R.S.Js by two wire ropes, which were spliced around solid thimbles through which pins in the top of the yokes were passed. These ropes were of such length that the spreader had ample clearance above the reinforcing bars projecting from the top of the column when vertical. A shackle was passed through a hole in the spreader beam and connected it to the return block of the crane.

A specially designed skid of mild-steel plate was clamped around the bottom of the column so that during lifting it would slide along on runners of mild-steel strip and so protect the concrete at the toe. A 5-ton hand-winch was used to control this movement. The column was gradually lifted to a vertical position with the crane luffing in stages, until it could take the full weight at a safe radius. The column was lowered gently on to a $1\frac{1}{2}$-inch-diameter pin set in the bottom of the foundation pocket, the fitting into a ferrule cast in the base of the column. Guys of wire rope were attached to the column on all four sides by means of a steel bracket passing through a slot formed in the column to receive the steelwork from the turbine-house floor. These guys were anchored to mild-steel loops which had been grouted into holes drilled in the concrete foundation. In each guy was a 10-ton rigging screw, so that once the column had been landed, it could be accurately plumbed and the column head adjusted to within $\frac{1}{8}$ inch of its true position. The final position was checked with theodolites set up on each line, sighting on marks at the top of the column.

Once the column had been accurately set, the foundation pocket was concreted and later the guys were removed. The column was then capable of taking all the stresses imposed on it under temporary conditions. The lifting of the main roof-beams turned out to be quite straightforward, although at first sight it seemed difficult because both cranes were in operation simultaneously.

The beams were cast across the turbine bay so that they were at an angle to the main system lines. For lifting, the cranes were precisely positioned so that the lifting points were on the same radius before and after erection, and the cranes did not need to luff during the operation. After lifting the beams over the top of the bars projecting from the column head, the cranes slewed in the same direction to bring the beams directly above their final position. By this means all lateral forces on the cranes would be balanced. The beams were lifted level without difficulty, and great care was taken to see that they were set in the correct position on both supports at precisely the same moment. It was regarded as essential

to eliminate any possible displacement of the top of a column caused by setting one end of the beam in position before the other.

The lifting attachments for the crane beams and main beams in the switch annexe were steel clamps, made so that they could be adapted to a beam of any size. These were clamped around the beams at the fifth points with hardwood bearing pads and hardboard packers at the sides and then attached to a bridle made of rolled steel channels. No difficulty was experienced in setting any of these units. The crane beams show an unbroken line nearly 300 feet long in the completed turbine house, and great care was necessary in setting them to ensure that the horizontal and vertical faces were truly in line.

The lifting of the roof units, which weighed 12 tons and were 26 feet long by 8 feet wide, provided another problem. In order to avoid over-stressing, a four-point lift was devised through a system of bridles. The unit was landed on four hardboard pads previously set in one plane on the main roof-beams. The unit was not designed for three-point support and it was not known whether the slabs would twist and crack if the four corners were not correctly positioned or there was not clearance along the top surface of the main roof-beam. All slabs were erected without accident, and the joints between roof slab and roof beam packed with cement mortar before the infilling concrete joint was made.

Site Organization and Programme

The order to proceed with the work was given on the 15th August, 1951, and the last roof unit was erected on the 28th January, 1952.

It was not expected to start concreting until the end of September, 1951, and the original programme was then to complete the gable-end and first bay in 4 weeks. After that, the erection was planned to proceed at an increasing rate until a speed of 2 weeks per bay was reached, which was felt to be the best likely to be attained. *Fig. 11* is a general view of the site 10 weeks after erection commenced.

The design and detail drawings were carried out in a sufficiently short time to make it possible to fabricate shutters, order bar reinforcement, and start concreting at the end of September as planned. In the meantime, the boiler-house basement floor was arranged as a bar-bending yard, the casting beds were levelled, and the concrete-mixing plant and 35-ton derricks were set up. The first units forming the gable-end were erected in the third week of October. After the erection gang had become accustomed to the methods involved, the building soon advanced quickly and, towards the end of the year, the maturing of the units was the chief factor controlling the speed of the job. Numerous concrete test-cubes were made when each member was cast. Tests of the concrete strength were taken at 5, 7, and 10 days so that each member could be erected as soon as it was strong enough. In each case, the resident engineer decided when a

precast unit was ready for erection. Towards the close of the job, the rate of erection was one 27-foot bay, including roof slabs, in a week. The concreting of the joints in the superstructure of the turbine house followed behind the erection at a convenient speed, for, with the feet of the main columns concreted in the foundation, the structure was stable and could take all the forces imposed on it. The concreting of the extensions on top of the D-line columns, which were to receive 360-ton loads from the boiler-house steelwork and the joints in the switch annexe were also made later, the latter being concreted with the floor slabs.

From the casting and erection point of view, the precast turbine-house roof slabs took the greatest proportion of the time, owing to the necessity of getting the long sides true, the accuracy required in setting them, and because they were cast in nests of three. It was necessary to erect all units in any one bay before travelling the derricks to the next bay.

The erection gang found that the accuracy of setting the various units and the various erection techniques adopted, meant that it was not possible to lift more than one major unit at a time. This meant that the derricks were not always fully employed, although the crane on F-line was generally busy lifting the numerous beams in the switch annexe which did not require so much supervision. The D-line crane was therefore used for lifting shutters, steel, etc., about the site but, undoubtedly because of the unbalanced nature of the job, it was not so busy as its opposite number.

It was found that it was necessary to work to very small tolerances in erecting columns, and all were checked by theodolites in both directions during erection. The methods of guying the columns and placing the beams meant that it was quite simple to adjust these units into their true position. This accuracy and the other aspects of the work entailed a greater degree of supervision than is usual on a building contract.

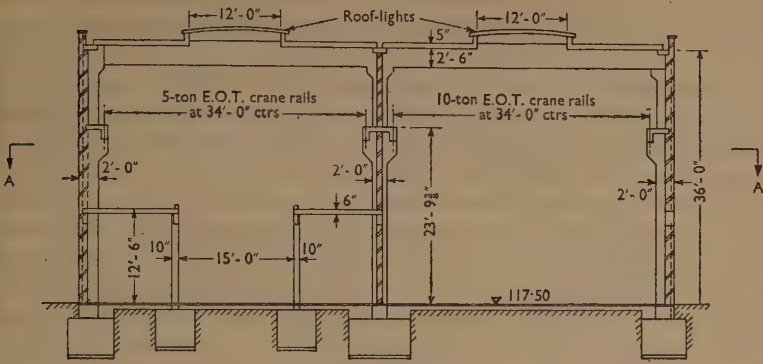
Workshops and Stores Block

The workshops-and-stores block is a separate building about 30 yards from the main power station. It is 75 feet wide, made up of two spans of 37 feet 6 inches with columns at 15-foot centres, the stores having seven bays and the workshops six bays. The height to underside of main roof-slab is 34 feet 6 inches. The stores part of the block has a mezzanine floor and a 5-ton travelling crane—and the workshops part has a similar crane of double this capacity. The columns are supported on mass-concrete foundations and the cladding was of 13½-inch brickwork which stiffened the building in both directions. A plan and cross-section of the building are shown in *Figs 12*.

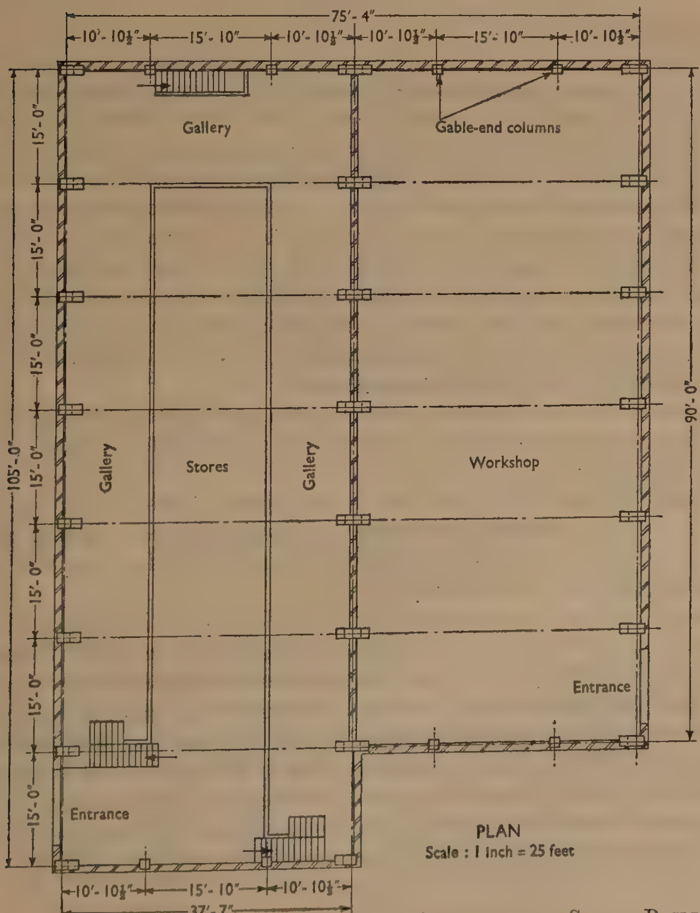
Design—General Principles

The principle elements, that is to say, columns, roof beams, roof slabs, crane beams, and internal floor beams were of precast concrete of 1:1½:3

Figs 12



CROSS-SECTION
Scale : 1 inch = 25 feet



PLAN
Scale : 1 inch = 25 feet

WORKSHOPS AND STORES BUILDING

mix, all of which was vibrated. Gallery slabs and external longitudinal beams were cast in situ in 1:2:4-mix and 1:1½:3-mix concrete respectively.

The frame of two bays is pin-jointed at the foundation, partially fixed at the eaves and partially continuous over the centre support, necessitating design in two distinct stages. Since the roof slabs were to be precast and no temporary props under the roof beams were contemplated in the unjointed condition, the columns and beams were, in the first place, designed as free-ended. Temporary guys were required until the joints over the columns at roof level were concreted, which was not done until after the roof slabs were fixed. The concreting of the joints converted the structure into a continuous frame and the design was completed to cover the effects of loading from the curved roof-light, the super-load, the wind, and crane forces.

Main Columns

The main columns are 35 feet 8½ inches overall in length and weigh 5½ tons. The external columns were 24 inches by 15 inches below crane-rail level, and 22½ inches by 15 inches above. The central columns were the same size at the base but 21 inches by 15 inches above crane-rail level. The main reinforcement was four 1-inch-diameter bars with two 1¼-inch-diameter continuity bars at each corner. A 1⅜-inch-diameter bar in hoop form from the centre column provided continuity for wind and crane loadings and live load from the roof. Brackets were formed as seatings for the crane beams and the main roof-beams, and the columns were designed for the resultant eccentric loading. Provision was made at the crane-beam brackets in the form of projecting reinforcement for connecting the crane beams, the joints being concreted-up.

The frames were tied at roof level with rectangular precast beams which rested on the column heads and were concreted into the principal joints with projecting bars.

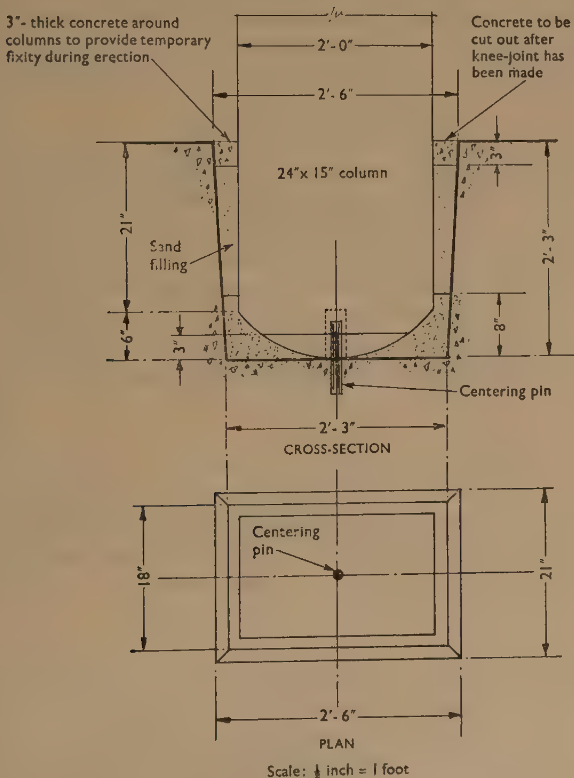
Serrated recesses were provided in the column faces to develop the shear and bearing values of the in-situ longitudinal beams. Slotted holes enabled four bars to be threaded through the column, lapping with the reinforcement of the in-situ beam.

The pin joint at the base of the columns (*Figs 13*) was made by filling the pocket in the foundation with sand, having first run a 3-inch thickness of grout underneath the column and around a centering pin similar to that in the main building columns. A 3-inch thickness of concrete at the top of the pocket provided temporary fixity during erection and was removed after the roof joint had been concreted. Mastic filling, run in between the column sides and the ground-floor slabs, completed the joint and prevented any moments being transmitted to the foundations. The maximum load carried on these columns was 100 tons.

Main Beams

The main beams were designed as free-ended for their own dead load and that of the roof units with continuity taken for loads imposed after the joints were made. A typical beam was 35 feet 10 inches long, 30 inches deep and 15 inches wide, and the principal reinforcement was six $1\frac{1}{4}$-inch-diameter bars. An extra 5 inches depth is obtained under the roof light.

Figs 13



JOINT AT BASE OF WORKSHOPS COLUMN

Projecting bars from the columns and ends of the beams lapped in the eaves joint and gave partial fixity at the corners. At the centre support the continuity bars were cranked under a continuous gutter which ran the length of the building. The weight of each beam was $8\frac{1}{2}$ tons.

Roof Slabs

A minimum thickness of 5 inches of solid concrete and a superload of 50 lb. per square foot was taken for the roof-slab units. Each slab was

14 feet 9 inches by 12 feet, and weighed $5\frac{1}{2}$ tons. Two lifting hooks were provided in each end, and extra reinforcement was added to take the bending moment between these hooks. The units were designed as free-ended and alternate bars projected at the ends to interlace with others protruding from the main beam. The space between the ends of the units was filled with fine concrete. The longitudinal joint between units was formed by a rebate in the adjacent edges, dovetailed in shape and this was filled with mortar after erection. Camber for drainage was formed in the main beam. Edge stiffeners on one side of the slabs provided kerbs for a curved glass roof-light similar to that in the centre of the main turbine house.

Crane Beams

The crane beams spanned 15 feet between the columns and carried comparatively light wheel-loads. The section was 22 inches deep by 12 inches wide overall, with a 5-inch-thick slab cast as a top flange and which provided a walkway alongside the cranes. Joints and fixings were treated similarly to the 70-ton-crane beams on a smaller scale. The main reinforcement in the 10-ton-crane beam was two $1\frac{1}{4}$-inch-diameter bars and two $\frac{3}{4}$-inch-diameter bars, and in the 5-ton-crane beam was two 1-inch-diameter bars and two $\frac{3}{4}$-inch-diameter bars. Compression reinforcement was not required and cross-carriage surge was taken by the slab.

Galleries

The short 10-inch-by-10-inch columns were cast in place in holes left in the foundation. Beams rested on the tops of these and a 6-inch-thick in-situ floor slab tied them together. Access stairs were cast in situ. The superload on the floor was $1\frac{1}{2}$ cwt per square foot.

Erection

The units for this block were cast on the floor of the building and erected, after the main building was complete, with a mobile crane fitted with an 80-foot jib. Erection was simple and straightforward, since the lifts were not particularly heavy, and was completed in 10 weeks (*Fig. 14*).

Figs 15 to *18* (between pp. 208 and 209) are further views showing the speed of erection which was achieved by the methods described in the Paper.

Fig. 15 shows the casting of the main roof-beams and columns. The second gable-end column is just being erected.

Fig. 16 shows the precast units awaiting erection after they had been precast on the basement floor.

Fig. 17 shows the placing of the crane beam on E-line columns and the first two bays of roof units complete.

In *Fig. 18* the last main roof-beam has been placed in position.

Advantages of Precast Concrete Construction

The use of precast concrete units in the manner described undoubtedly raises many points of interest which should receive serious consideration. The economy in the use of steel is, at the present time, of great importance, and the weight of steel actually used is of great interest. The total weight of steel used in the frame of the turbine house only was 176 tons, or 0·27 lb. per cubic foot of building; when considering this, allowance must made for additional loads carried—360 tons per column from the boiler house on the one side and a multi-storey switch annexe on the other. The weight of steel in the complete precast frame was 330 tons (0·36 lb. per cubic foot of building) and this may be compared with the estimated figures for the original steel frame of approximately 900 tons (1·00 lb. per cubic foot of building). For the complete building (reinforced-concrete frame, including all precast and in-situ floor and roof slabs) the weight of reinforcement used was 450 tons (0·48 lb. per cubic foot of building).

There is no great difference between the weight of steel in the permanent structure and that in a reinforced-concrete structure cast in place in the normal way. However, it is the Authors' experience that with in-situ construction, large amounts of steel are used in the temporary works. A common procedure is the construction of a structural-steel travelling shutter running on the permanent crane-beams to support the turbine-house roof shutters, and this would require more than 50 tons of steel—more than 10 per cent of the steel in the permanent structure.

The weight of steel in the workshops and stores, a type of building more frequently encountered, was 37 tons total (0·29 lb. per cubic foot of building) of which three-quarters was in the precast frame.

Another great gain was the time saved. The frame, including the necessary alterations to the foundations, was substantially complete in less than 6 months from the time of placing the order. This compares favourably with the time which would have been required for obtaining, fabricating, and delivering the steelwork.

Other savings result from the reduced amount of maintenance necessary. The steelwork, if not encased in concrete, requires frequent painting, and it should be noted that at Acton Lane, because of the high quality of finish of the precast concrete members, no further finishing such as plastering was considered necessary.

The initial cost of a reinforced-concrete frame is generally less than that of a steelwork alternative, but the amount of saving is affected very much by the type of structure and the amount of repetition of precast units.

An important consideration is the great accuracy with which the construction must be carried out. Shutters must always be true to line and level, otherwise meeting faces do not marry, and tolerances in erection must be kept down below $\frac{1}{8}$ inch, not only at ground level but also 60 feet up in the air. This control requires a high standard of supervision.

Conclusions

Acton Lane turbine house is only one of several jobs of this type for which schemes in precast concrete have been prepared, but it serves as a very good example of what can be done.

The Authors find that if the weight of the members can be kept within the capacity of lifting appliances such as the derricks described, the speed of erection is excellent and a simple and cheap job results. The frequent use of a shutter for a small unit is obviously cheaper than obtaining only a few lives from that for a much larger one. However, schemes have been prepared where this advantage is not present but they are without doubt fully competitive with the steelwork alternative.

In one example, which was built prior to Acton Lane, a large engineering workshop was erected with concrete units precast on the site, but since only a light steel roof was required in this case, the roof steelwork, covered with bituminized metal sheeting, was retained and erected on the concrete frame. In a much later example, a scheme has been completed for a similar structure to Acton but with main column centres at 112 feet, and with system lines at 70-foot centres. In this, nearly all beams are of prestressed concrete, and the crane beams support a 150-ton travelling crane over a span of 70 feet at a height of 60 feet above ground level and weigh more than 80 tons each.

The Authors consider that the method described of concreting the main members of a structure on the site has several advantages over a structural-steel frame, and believe that Acton Lane has shown how this can be done cheaply, quickly, and with great savings in steel and skilled labour which are at present scarce.

Acknowledgements

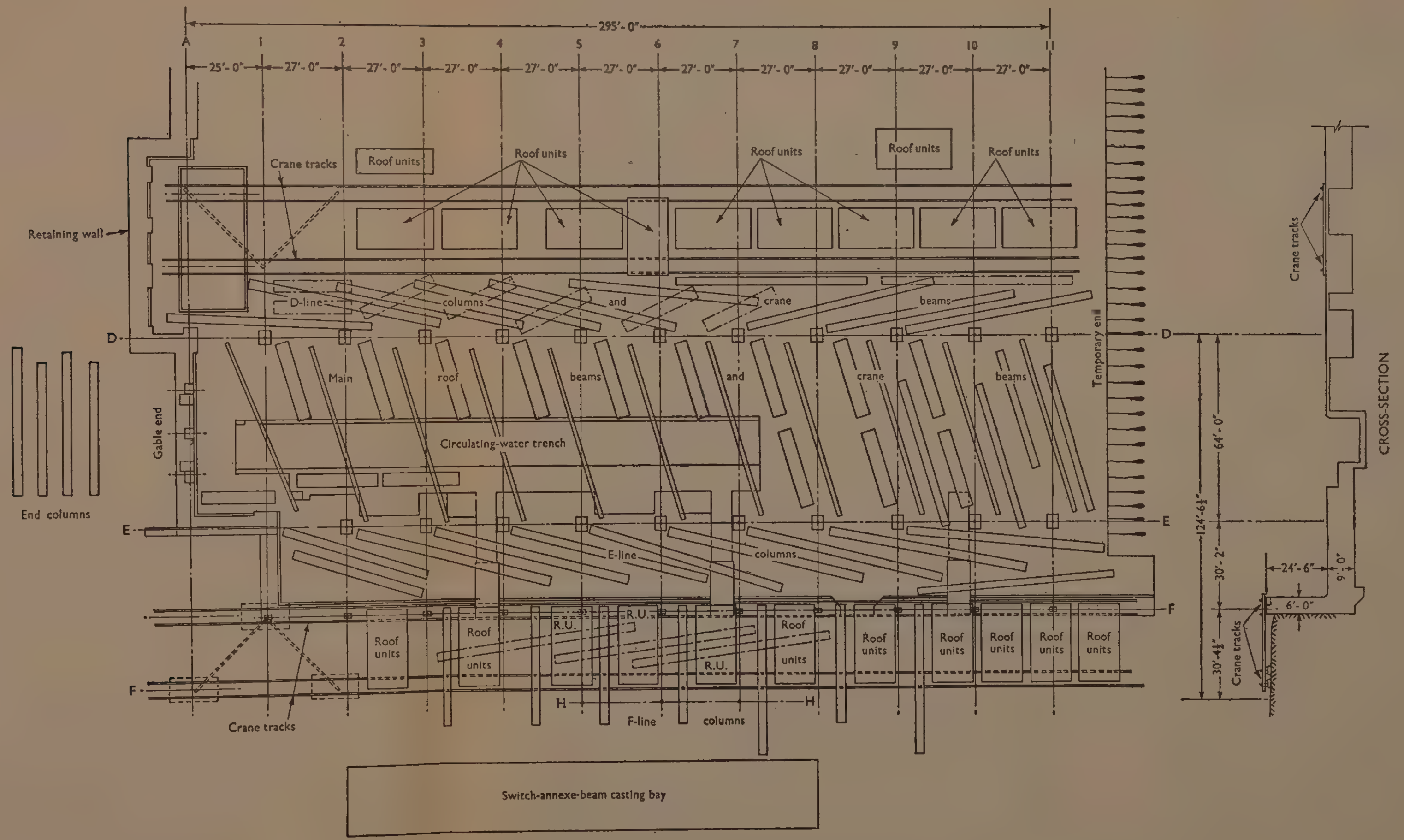
The Authors wish to thank the British Electricity Authority (London Division) for permission to present the information contained in this Paper.

The Divisional Controllers responsible for the erection of the station were Mr J. N. Waite, C.B.E., M.I.E.E., and later Mr H. V. Pugh, M.I.Mech.E., M.I.E.E.; the Chief Generation Engineers (Construction) in charge were the late Mr A. Pollitt, M.I.Mech.E., A.M.I.E.E., and later Mr C. A. Clench, M.I.Mech.E. The Resident Engineer on the site for the British Electricity Authority was Mr F. J. McQueenie.

The original scheme was conceived by the Contractors, Sir Robert McAlpine & Sons Ltd, to whom Messrs Brian Colquhoun & Partners were Consulting Engineers for the design of the reinforced concrete work.

The Contractors' Chief Engineer on the site was Mr J. C. Holohan, B.E., A.M.I.C.E., and the Works Manager was Mr J. J. O'Neill, B.Com.

FIGS 8



PLAN

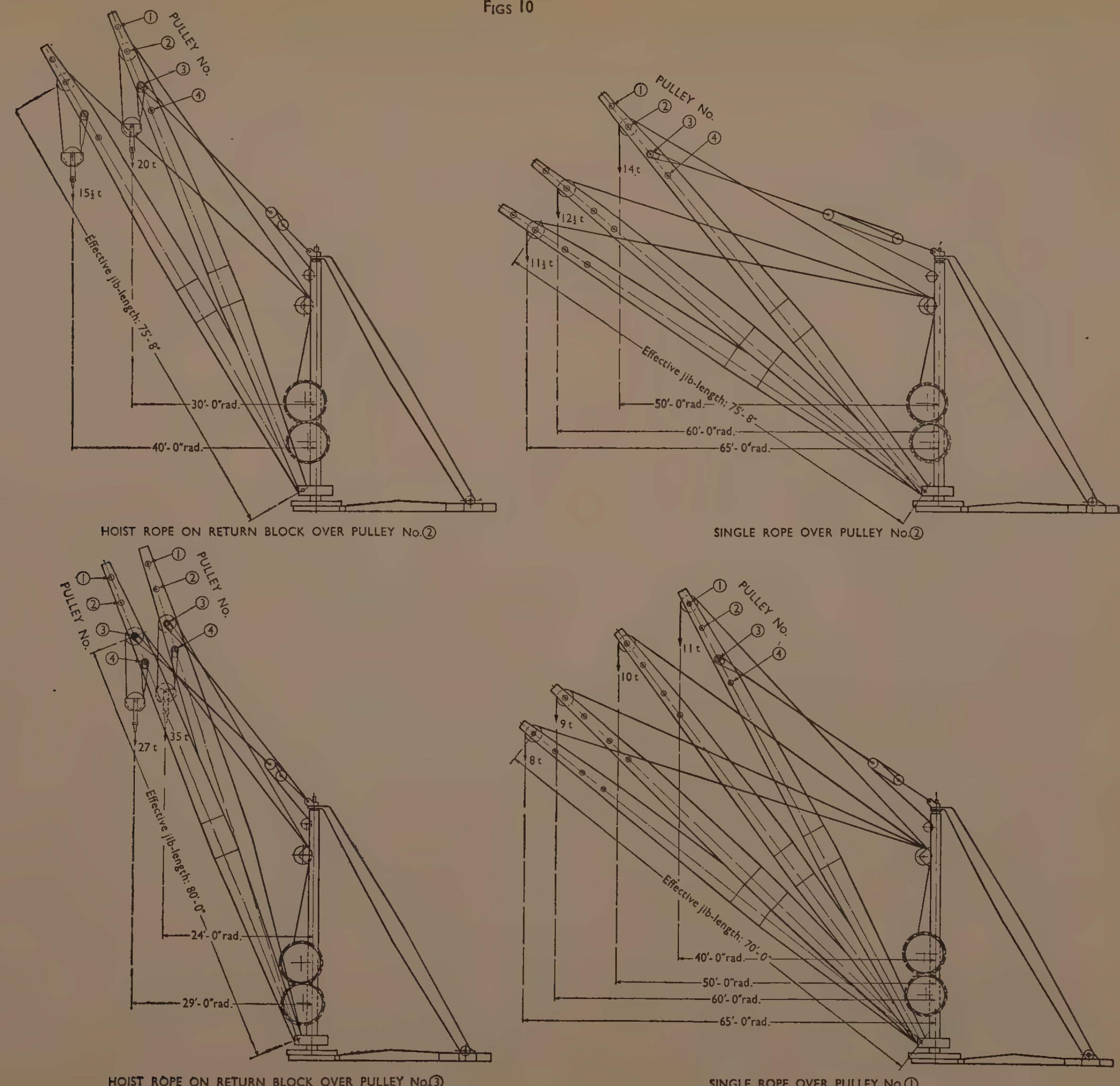
PRECASTING LAY-OUT

J. A. DERRINGTON and A. G. S. LANCE

WILLIAM CLOWES & SONS, LIMITED : LONDON

THE APPLICATION OF PRECAST CONCRETE TO THE CONSTRUCTION OF ACTON LANE "B" POWER STATION

FIGS 10



DIAGRAMMATIC DETAILS OF DERRICKS

SAFE WORKING LOADS

Pulley No.	Distance between jib pins	Radius						Remarks
		29'-0"	30'-0"	40'-0"	50'-0"	60'-0"	65'-0"	
		Tons	Tons	Tons	Tons	Tons	Tons	
3	70'-0"	27						With return block
2	75'-8"		20	15½				Do. Do. Do.
2	75'-8"				14	12½	11½	Single rope
1	80'-0"			11	10	9	8	Do. Do.
3	70'-0"	Radius 24'-0", safe working load 35 tons						With return block

Messrs Lenscrete were the sub-contractors for the roof lantern and lights.

The Authors also wish to thank the following for their assistance in preparing this Paper :—

The Staff of the British Electricity Authority (London Division), Mr R. Goymer of Messrs Brian Colquhoun & Partners, and Messrs H. F. Rosevear (who was responsible for the preliminary scheme), A. P. Mears, B.Sc., and P. J. Kershaw, B.Sc.(Eng.), of the Contractors' staff.

They would also like to record their appreciation of the great co-operation between the staffs of the Consulting Engineers and Contractors, which enabled the job to proceed with great speed and efficiency.

The Paper is accompanied by ten photographs and nine sheets of drawings, from which the half-tone page plates, folding Plates 1 and 2, and the Figures in the text have been prepared.

Discussion

The Chairman, when proposing the vote of thanks, said that the Authors had described a job which perhaps did not differ very much from many which had been done before, except in respect of size and speed of erection. Evidently its size had given rise to some problems which were difficult, but which had been successfully overcome. Some of the advantages of the precast system were mentioned towards the end of the Paper, and he would have included among them the fact that it was easier to work on a work-bench than at the top of a ladder ; in other words, it was easier to place and keep the steel reinforcement in the right place at ground level than to do so anywhere else. Was he not right in thinking that a precast member could fairly be claimed to be an article which was superior to a member cast in situ ?

Professor A. L. L. Baker said that the Paper described what he considered to be a very bold step forward in construction methods. It had shown that precast construction with heavy members paid and that it could be done in reasonable time. It competed with structural steelwork, and it had the very great advantage that it was proof against the weather. The danger with much concreting work was that when very good concrete had been poured a shower of rain might spoil it. With precast concrete one could be sure of factory-quality concrete and should therefore be able to work to very much higher stresses.

Immediately after the war there had been a good deal of talk about the possibility of this kind of construction for multi-storey building. Professor

Baker showed a slide illustrating a proposal for a tall framed building in which the precast units weighed no more than 3 tons and consisted of parts of the framework and whole sections of the floor. Suitable splices and joints had been worked out. At the time, many practical men had maintained that it would not pay, and that it was much cheaper to use small precast units or to use cast-in-situ construction. That argument was still put forward today, but the present Paper had clearly shown that it was complete nonsense and that it did pay to use precast units, even up to 30 tons in weight. It might be that, in the case in question, the particular type of construction was suitable for the heavier units, but if it was a practical proposition with a structure of that kind it should be a practical proposition with multi-storey buildings, and he would like to see some buildings in London constructed in that way. Development had, he knew, gone a long way towards it, but he thought that it should go further and defeat the often-used arguments against concrete, which were that weather could militate against the quality of the concrete and that it was difficult to ensure adequate supervision of concreting work being carried out many feet above ground level. It was, on the other hand, possible to rely on precast concrete made under what were really factory conditions at ground level.

Professor Baker had been very interested in the base of the columns and the way in which the load had been transferred to the base. It was quite a simple and obvious way of doing it, and yet it was one of those steps forward which many people would be reluctant to take. Only 6 months previously, during the steel shortage, a very eminent architect had told him that he could not possibly continue his construction in reinforced concrete instead of structural steel, because he had put steel grillage foundations into the base and had no projecting splice bars. There was a prejudice and fear in the minds of many people that there was only one way of joining a column to a footing, and that was with splice bars. He had been very pleased to see, therefore, that the new method described in the Paper had been tried and found to be successful.

Turning to the question of design, he asked if the Authors would confirm whether it was the intention for the lateral wind stresses on the turbine house to be spanned back longitudinally, using the roof as a stiff horizontal beam, to the end bays. That was, Professor Baker thought, a good way of dealing with wind stresses. Engineers were so accustomed in their design exercises always to have portal frames in reinforced concrete that they were apt to forget when columns were long that it did not necessarily pay to do that, and that it might pay to do what the Authors had done and have the horizontal member mainly simply supported, with a slight amount of continuity, and to take wind stresses or horizontal forces by spanning them back through the roof as a horizontal beam to stiff walls at the end.

The crushing stresses for the concrete cubes were about 5,560 and the permissible design stresses 1,500 lb. per square inch. What would the prestressing protagonists have done about that? It seemed to him that

there was an anomaly there, in that those who used reinforced concrete had by regulation to provide a load factor of nearly 4, and about 5 after 3 or 4 months, whereas the man who used prestressing was permitted to use a factor of about 2·5 or 2·75 against failure by the concrete. Would the Authors comment on that point?

Could the Authors supply a few more details of the roof joints and the joints in the workshops and store? He felt that those joints were extremely important and that to have more details of them would be very valuable to engineers. It was extremely difficult, he thought, to design a satisfactory joint in precast construction. It was necessary to splice the steel properly and to arrange that the local concrete could be easily placed and that it would be dense and join up with the other concrete. Moreover, the two members had to be fitted together exactly, which was often difficult without some seating arrangement. The more information that could be given on precast joints which had been used successfully, the faster would be the progress with that type of construction.

Mr Brian Colquhoun agreed with the Chairman's remarks referring to the great accuracy which could be obtained by carrying out reinforced concrete work, so to speak, on the work-bench rather than 60 feet up in the air. There was no doubt that it was far easier to place the steel accurately and see that it was kept in position when the work was done on the ground; it was easier for the man who placed the steel, and also for the man who carried out the inspection to see that steel was in position at the time when the concrete was placed. That alone was an advantage in reinforced concrete, but what had been done in the present instance went a good deal further, because reinforced-concrete beams and columns had been put up which weighed about 30 tons each. That in itself involved the use of plant which not every contractor possessed, and therefore the attempt to carry out a scheme of the kind in question necessarily involved having the right plant to do it.

The speed with which such a job could be carried out, and the economy which was effected, was dependent upon the very closest liaison and co-operation between the designer and consulting engineer and the contractor who had to carry it out. Too often was a job, not necessarily in reinforced concrete, designed without any reference to the way in which it was going to be carried out or to the contractor who was going to do it.

Mr H. Wingrave Newell observed that Mr Colquhoun's remarks on liaison and team-work were particularly apt in view of what he himself was about to say. On the job in question, a very considerable amount of team-work had been evident even at the drawing-board stage. He himself had been responsible only for the natural lighting of the structure—the roof lighting— but even so the amount of calculation and the number of drawings and alterations to the original design in order to accommodate that had been quite considerable.

The problem of the roof lighting consisted of four major problems.

There was first the question of providing sufficient natural illumination, which in itself was quite a major problem. It was so often solved by drawing a number of rectangles on a roof span and letting it go at that, but in the present case a considerable amount of thought had been given to it. First of all, a candle-power factor had been decided on, and, using Technical Paper No. 28 of the Department of Scientific and Industrial Research, curves had been drawn and eventually the intensity diagrams had been produced.

The second problem had been to make sure that the construction was sufficient to provide the lighting required and also safely to support the superimposed load of 50 lb. per square foot. Thirdly, the problem of condensation needed consideration, and fourthly it was essential that the construction could be economically erected.

The problem was therefore concentrated on the centre panel, where the monitor had been introduced for two reasons. The first was in order to maintain a cross-current of ventilation by controllable ventilators set in the sides of the monitor. The monitor itself was raised sufficiently high so that temporary bridging beams could be erected across the opening and the lights cast from those bridging beams. That had been a big problem, because the ground floor was about 70 feet below that level, and the cost of scaffolding would have been very considerable. The monitor itself did not provide a sufficient intensity of illumination, so additional lighting panels had been incorporated in the roof itself, where the problem of condensation arose because there were no means of introducing a cross-current of of ventilation.

Those lights were of double construction. They were sealed on the soffits with "Rubazote" gaskets in such a way that the thermal transmission was approximately halved and condensation was not likely to occur. The problem of construction there had been difficult, because there was no good way of introducing within the flat roof any form of bridging girders, and consequently the two long stretches of lenses on each side of the main monitor light had been sub-divided into small areas and precast. They had been precast, lifted to the roof, and the soffit glazing placed, and then the units had been placed in position. He was grateful to the contractors and engineers associated with the work for the opportunity of co-operating with them in the early stages in the design and construction of those lights.

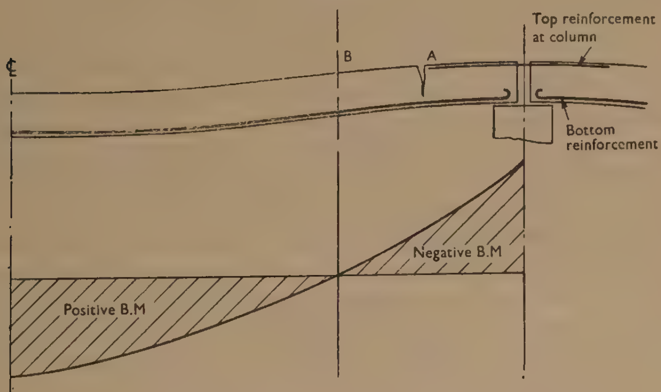
The method of lighting was not new; they had been employed to a very considerable extent in France. Mr Wingrave Newell showed three lantern slides illustrating similar types of work, including the roof lights of Bankside Power Station (slightly larger span than Acton Lane) and the lighting of the Gare de Versailles, which had been carried out 27 years ago.

Dr T. P. O'Sullivan said that the technique of precast concrete construction was one of the branches of engineering in which he believed Britain had a lead over the United States. It appeared that there were

still differences of opinion on the subject and various schools of thought, but he thought that precasting should have made more rapid progress than it had done so far. Did the Authors agree ? It ought to make more rapid progress now, because one of the important features in connexion with the work described in the present Paper was the speed with which the work had been carried out. The details given in the Paper disclosed an astounding performance in that respect. Could any further information be given about the relative cost in comparison with other forms of construction, and could comparisons be made with in-situ concrete and steel ? He was not, of course, asking what the contractors' costs were.

There was one design aspect about which he was a little concerned, and he thought that it was a matter which required careful consideration. It appeared that most of the beams had been designed as free-ended, and

Figs 19



that some reinforcement had been introduced to permit a certain amount of continuity. He had always had the impression that a beam was either a free-ended beam and deflected freely or else was continuous and properly restrained. *Figs 4* showed, of course, that there was a considerable amount of top steel which would undoubtedly cater for a considerable negative moment. He thought, however, that where precasting was used in such a way in the future, difficulties should be guarded against. When two freely-supported beams rested on the same column there was a tendency for a crack to appear between the upper ends of the abutting faces, immediately over the column. If there was some provision for continuity where there was a system of that kind, and if it did not crack, there was a hogging bending moment over the column ; if that had not been designed for, it would appear that there might be some condition of over-stress. What were the Authors' views on that matter ?

Figs 19 showed what might happen on a precast job. If a considerable

amount of top steel had been provided and carried right over the limits of the negative moment there would be no trouble, but if the top steel was carried only to the point A and the negative moment finished at B a crack might occur between A and B and there was nothing to prevent failure.

The Authors appeared to have used asymmetrical reinforcement very extensively, and there was a reference on p. 206 of the Paper to a saving of 30 per cent. Dr O'Sullivan had at one time been particularly interested in the subject of the economic distribution of reinforcing bars and had presented a Paper [1] on it. In view of the Authors' statement that a saving of 30 per cent had been effected, it would be interesting to know how far their development agreed with or differed from the ideas expressed in that Paper.

Mr P. G. Boosie said that he imagined that in the filling of the joints there must be some tendency for shrinkage to occur, and he would like to ask whether that presented a serious problem in the precast work. It occurred to him that in a horizontal joint between two vertical columns there was bound to be considerable direct loading. He did not think that that occurred seriously in the particular instance under discussion, but he could think of another contract executed by the same firm in Kensington where vertical columns had been joined together, and it would be of some interest to know the condition after the placing of the in-situ concrete.

Had the two cranes used at Acton Lane been specially designed for the job or were they fairly standard pieces of equipment.

Mr E. F. Humphries referred to *Fig. 5*, which showed the joint at the junction between the two crane girders. Was it a fact that concrete had been induced to the extreme bottom of the tapered splay at that point? It was essential that that should be done to transfer the load to the column. Moreover, the main crane beam in that type of construction was part of the structure which took a considerable amount of load. The overhead crane was used a great deal, and with continual use and the crane surge which took place in both the longitudinal and transverse directions it seemed that a more positive junction of those crane beams than by the well-known method of interlocking reinforcement with in-situ concrete should have been provided. Had the Authors considered a positive junction by means of a scheme which had been adopted by others, namely steel connectors left at the ends of the units and bolted together, and bolted, perhaps, to steel plates from the columns, or, alternatively, had they considered welding of the reinforcement to provide a far more positive fixing than had actually been given? Had those points been considered, or would they be considered in future if a similar job were carried out?

In many power stations it happened very often, though not always, that the boiler-house structural steel framework was erected before that of the turbine house. In the work described in the Paper, there was the very

[1] "The Economic Design of Rectangular Reinforced Concrete Sections." J. Instn Civ. Engrs, vol. 32, p. 175 (Apr. 1949).

great advantage of a completely free site; the foundation slab for both turbine house and boiler house had been completely ready, and the contractors had been able to use the whole area for their casting shop and also to use the space of the boiler house for one of the erection cranes. If structural steelwork had been in course of erection for the boiler house, what would have been the situation? The casting area would have been severely curtailed and one crane could not have been placed in that position. How would the Authors meet that difficulty should it arise, and, in the case described in the Paper, might it have altered the whole set-up of the job? Possibly the Authors would have used alternative methods of erection, perhaps with vertical masts or "sticks," instead of derricks. What was the maximum weight which the Authors considered could be erected in such circumstances, bearing in mind the necessity for accuracy of working which they had so clearly expounded in the Paper?

Mr Derrington, in reply, said that although there had been few speakers, their comments had ranged over a wide field. The Chairman had referred to improved working conditions on the ground. The advantages of being able to get at a unit which was being concreted with so much ease were apparent. Naturally a higher grade of concrete was obtained; it was much easier to vibrate the concrete, and it was regrettable that at the moment the working stresses were limited to the same values as would normally be used if it were placed 60 feet off the ground or in any other position. Advantage could be taken of the clause in C.P.114 which allowed the working stresses to be increased by 10 per cent provided the cube tests justified that, and perhaps in the present state of knowledge that margin was suitable. Another advantage of precast work was the much better finish to the units which was obtained. At Acton it was understood that the client did not consider it necessary to give any surface finish to the members; it was considered that the concrete had such a high standard of finish that plastering was unnecessary, and it would probably be quite difficult to get any finish to stay on that concrete surface,

The Chairman had also referred to the advantages of quicker working and greater reliability in the position of individual reinforcing rods, which, of course, with a higher stress and other factors, was most important. With reference to his remarks on the comparative speed of construction in England and America, anyone who thought that, in a period of 7 weeks after getting the word to go, a job such as had been carried out at Acton could be finished, they were welcome to try. The time taken for the job included starting from scratch with the architectural lay-out; it also included not only the design and detailing of the individual members, but also a great deal of discussion aimed at making certain that the units could be picked up with the plant which was available, and that they could be fitted into position.

Professor Baker had referred to the working stresses in the concrete and also seemed to imply that that the larger the unit the better the

form of construction. Mr Derrington agreed wholeheartedly—perhaps he had to, having once been his pupil. He felt that if it was possible in a building to prefabricate or precast individual units in as large a piece as possible, one could rely more fully on the individual joints and cut down the cost and the time taken for erection, as well as gain several other advantages arising from the more homogeneous method of construction.

He differed from Professor Baker on the question of joints. His particular interest was the blending of steelwork construction with concrete. It seemed to him that when dealing with a new medium—in other words, precast reinforced concrete—consideration should not necessarily be confined to the methods used for the joining of prefabricated steelwork units. To his mind, it was wrong always to think of joining concrete units with gusset plates, cleats, bolts, and steel sections. Reinforced concrete itself was a far more flexible medium—it would conform to any desired shape. Careful attention had to be given to the question of joints, but it was not necessary to follow in the footsteps of steelwork designers.

So far as wind stresses were concerned, the switch annexe alongside the main building had been of considerable help, because the wind force was taken by panel walls right across the building at alternate column positions, and in addition there was a brick panel wall across the gable end, so that the roof slab was used as a horizontal girder to distribute those forces along the length of the building and to pass them on to the stiff diaphragms which occurred every 54 feet. Generally speaking, he felt that the construction of the brick panel walls had resulted in a very stiff building.

Professor Baker had also mentioned the question of concrete stresses. It should be pointed out that the concrete cube strengths referred to were average strengths which were not always attained, so that a figure of 5,500 lb. per square inch for the average cube strength would not have so much in reserve if a higher working stress was used. The question of the relation between the maturity of the concrete and the actual strength of the individual units when a load was placed upon them was now, however, becoming the subject of more general discussion.

On the question of joints, in places there might be a tendency to become a little over-enthusiastic in making certain that reinforcing bars were adequately lapped. It seemed to him that a better joint would often result if one spared the steel and made certain that the concrete could go in just that little more easily.

Mr Colquhoun had referred to co-operation between the contractor and the engineer. It would certainly never have been possible to start casting concrete units only 3 or 4 weeks after receiving the order without the exhaustive discussion which had taken place between Mr Lance's office and his own.

Mr Wingrave Newell had given a few golden rules on how to design roof lighting. Mr Derrington thought that on the job in question the pro-

blem of blending an adequate intensity of lighting with a very pleasant appearance had been successfully overcome. The questions of ease of construction and the provision of adequate clearance for construction had been adequately dealt with only after a great deal of preliminary discussion at the earliest stage.

Dr O'Sullivan had referred to the lack of progress in the development of precasting used on a large scale. That was probably partly attributable to the use of prestressed units rather than of normal reinforced concrete units having stolen the public eye, but it was also in large measure due to the limitations of the plant which was at present in common use. It seemed to Mr Derrington, however, that there was in some quarters an unwarranted prejudice in favour of precasting on every possible occasion. Precasting had specific advantages which had been outlined in the Paper, but it had some difficulties also. A job of the size of that at Acton had to be put together with exactly the same accuracy as an equivalent steel structure, and it was no good casting units on the ground and then trying to put them together unless a great deal of very careful supervision had been given to the casting of the individual members, and an even greater amount of care given to the manner in which they were erected and fixed in position.

The question of relative cost always arose, and probably the best thing to say was that if the individual rates were quoted they would mean nothing, because he believed that steel had increased in price by 40 per cent in the past 2 years; cement prices were falling, but the cost of labour varied considerably. Individual costs, therefore, did not mean a great deal. Towards the end of the Paper, however, reference was made to the relative amounts of steel which would have been required if the building in question had been built in structural steelwork, and to that required with normal reinforced concrete construction. It must be borne in mind that the individual size of the members did not vary greatly between a steelwork structure and a reinforced-concrete structure, and that if the original steel structure had been covered with concrete as a form of fire-proofing a very close approximation could be made to the cost of it by considering the weights of steel. He did not suggest that, because 60 per cent of the steel was saved, 60 per cent of the money would be saved, but it should be borne in mind that with a structure of the type in question the weight of steel was some guide on whether or not it was economic.

On the question of continuity, Dr O'Sullivan had raised the point of the main roof-beam. It should be made clear that in erecting the structure consideration had to be given not only to its final finished state, where the building was stiffened by the cross-walls in the switch annexe, but also to its position when joints between the roof beams and the columns had been made but no stiffening walls had been constructed. Care had had to be taken that, in its temporary state as a portal frame, the structure should safely take the stresses which would be imposed on it by wind. Some of

the top steel in the main roof-beam had been put in to increase its strength. It was compression reinforcement and, though it served very well for lapping the bars round the outside of portal-frame joint, it had been placed there primarily for compression purposes. The asymmetric reinforcement in the columns had been found of great advantage, especially in those columns which were rather small in size, where the actual centre of gravity of the equivalent section had been used in reducing the eccentricity of the load and consequently the bending stresses.

Mr Boosie had raised the question of shrinkage in the joints. In the job at Kensington to which he had referred the columns were all cast in situ, and only the beams precast, so that so far as the joints in those columns were concerned it was normal in-situ construction. The question had been raised, however, about the relative difference in stresses imposed on the in-situ concrete in the joints and the precast concrete in the units of a precast concrete structure. A great deal of thought had been given to that point and it seemed to Mr Derrington that there was much still to be found out about it. At present, it was ascertained that the limits, between which the stresses would lie, were acceptable. On first consideration, it would seem that there was a difference in stress between a member which was carrying part of its load when it was erected and another section of the structure which was added after that initial member had been partly stressed. What that difference was it was very difficult to state. It might well be that the in-situ concrete filling had, by reason of its own shrinkage, a small tensile stress, and also imposed a compressive stress on the precast part of the column. However, the final load added to the whole section would overcome any slight tension which might occur in the in-situ filling, and the result was a section which might well be stressed evenly, or in which the stresses in the in-situ filling and in the precast portion did not differ by a great deal. Much investigation, however, had to be done on that point before one could be quite certain what took place.

The derricks were not standard, but had been adapted from existing derricks with the co-operation of the makers.

Mr Humphries had pointed out that great care had had to be taken to ensure that fine concrete filling underneath the splayed end of the crane beam was placed and well compacted. That was true, because at that point the whole of the bearing of that beam was passed through to the column. It was generally done by caulking with a dry concrete, and the other part of the in-situ joint could then easily be placed. In all, the joint was approximately 24 inches by 12 inches in section, so the problem was no more formidable than that of concreting a plinth of that size 3 to 4 feet high. To make certain, however, that that beam passed its load properly to the column, the joint at the seating had to be well caulked with dry concrete.

With regard to the erection of the building at Acton on a completely free site, the boiler house side had been available for casting but, as pointed out in the Paper, the planning of the casting yard was a very difficult and

highly involved problem, and in some cases it could determine the methods to be used in actually designing the structure. However, that difficulty had been overcome where the advantage of the boiler house site being free was not enjoyed, and as Mr Humphries had pointed out, other tools for erection could well be used.

On the question of what sizes of members could be precast, Mr Derrington understood that at Moelsbroek airport in Belgium precast units of 165 feet span weighing 270 tons had been cast on the ground and erected to the design of Professor Magnel. The precast units at Acton Lane were much smaller than that.

Mr Lance, in reply, said that the ideas expressed in Dr O'Sullivan's Paper had been studied and had proved very helpful, when designing the main columns, in eliminating much of the donkey work in arriving at trial sections. Further, with respect to Dr O'Sullivan's point as to continuity of the main roof-beams, much of the top reinforcement adjacent to the supports had been determined by the cantilever bending moments set up when the beam was being slung into position and supported by slings.

Correspondence on the foregoing Paper is printed on p. 340.—SEC. I.C.E.

PUBLIC HEALTH ENGINEERING DIVISION MEETING

24 March, 1953

Mr G. M. McNaughton, Member, Chairman of the Division, in the Chair

The following Paper was presented for discussion and, on the motion of the Chairman, the thanks of the Division were accorded to the Author.

Public Health Paper No. 6

"Controlling Factors in the Choice of Sewage-Treatment Processes"

by

William Fillingham Brown, B.Sc.(Eng.), M.I.C.E.

SYNOPSIS

In an introductory section the Author discusses briefly the effects on rivers, and other natural bodies of water, of the discharge into them of untreated sewage arising from the general adoption in urban communities of the water carriage system. The nature of the problem of adequate sewage treatment to prevent pollution of the receiving body of water is examined, and the usual criteria, that is to say, the suspended-solids content and the biochemical oxygen demand of the effluent, are stated. Reference is made to the Royal Commission Standards and their application.

The processes now in general use for sewage treatment and for sludge treatment and disposal are listed. The factors determining the choice of these processes are divided into two categories, primary and secondary, as follows :—

Primary :— (1) The standard required for the effluent.
(2) The character of the sewage to be treated.
(3) The size of the system (or quantity of sewage to be treated).
(4) The ruling economic factors.

Secondary :—(1) Climatic conditions.
(2) Conditions relating to the site of the treatment works, including surface, and subsurface conditions, and amenities.
(3) Materials and labour available for construction, operation and maintenance of the treatment works.

The remainder of the Paper is devoted to detailed discussion of all these factors and the ways in which they affect the choice of treatment processes.

Introduction

The maintenance of public health in urban communities depends in a very large measure upon the provision of an abundant supply of pure water and the removal, under sanitary conditions, of domestic and industrial waste products. The adoption of a piped water-supply in even the smallest communities requires some means of removing and disposing of the water after it has been used, or insanitary conditions soon arise. The

most economical and efficient way of achieving this is by the provision of a network of inter-connected drains and sewers, with connexions from all dwelling houses, business premises, and factories, through which all liquid and some solid wastes (including all faecal matter) may be conveyed to one or more points of disposal. This system, known as the " water carriage system," though used to some extent in a primitive form in several ancient civilizations, was developed in modern times by British engineers. It has proved so successful on sanitary as well as on economic grounds, that its supersession by any other system is now unthinkable. In fact it may be ranked in importance with the invention of the steam engine and the dynamo, or even with the more fundamental invention of the wheel itself in the tale of man's achievements in his advance towards better material conditions of life.

The general application of the water carriage system in any community results in the concentration at one, or at most a few, points of the " sewage," a highly complex mixture of water with organic and inorganic wastes, some in true solution in the water, some in suspension, and some in colloidal form. The eventual destination of all sewage is the watercourse or other body of water which constitutes the natural drainage channel or receiver of the area, and one obvious method of disposal of sewage is to allow it to discharge, without previous treatment, into such channels or receivers, a method still largely practised in many parts of the world, and originally in common use in England. In the case of a river, unless the quantity of sewage is very small in relation to the flow of water in the river, the method merely extends the sewerage system to include the river channel, and leaves the disposal of the sewage to Nature, a task which Nature willingly accepts and carries out quite efficiently up to certain well-known limits, but beyond which the work is completely given up, almost it would seem in disgust, and the river is abandoned to its consequent foulness. In more scientific terms, a comparatively small proportion of sewage, by reason of the oxygen demand of its complex and chemically unstable organic constituents can reduce the dissolved oxygen content of the water of a river below the minimum necessary for the support of animal and vegetable life, and so below the limit for the maintenance of " self purification."

Many rivers in England were reduced to this sorry state during the nineteenth century as a result of the rapid increase in the size of towns and of industry generally, and the introduction of piped water-supplies and systems of sewerage ; some have even yet by no means recovered from the ill-treatment they have received, and still receive, from the continued discharge of sewage and industrial wastes into them. The health of the rivers was, in fact, sacrificed to the requirements of industry and in order to safeguard human health and well-being.

Where the body of receiving water is very large, for example, lakes, large estuaries, or the sea itself, the effects of discharging the sewage from

even very large communities may be hardly noticeable. Thus, for many years, the cities established on the Great Lakes of the North American continent continued to discharge all their sewage into the lakes after little or no treatment, and also to derive the whole of the water they required from them. It is only in recent years that treatment of sewage has been found necessary, so great is the dilution afforded by the body of water in the lakes. The sea provides an almost infinite dilution, and apart from tidal conditions which may result in the fouling of the foreshore, or the spoiling of bathing beaches, will continue to provide the most economical form of sewage disposal for most seaside towns. For most inland communities in Great Britain, however, the discharge of their untreated sewage and industrial wastes into rivers would result in gross pollution of those rivers, and eventually all normal animal and vegetable life would be destroyed.

Sewage Treatment—The Problem

It will be seen from the foregoing, therefore, that the problem of sewage treatment resolves itself into the application of suitable means of preventing the pollution of the watercourse or body of water into which the sewage would eventually find its way by natural drainage. The only practical alternative is complete absorption into the subsoil, with or without the addition of various crop growing techniques—a method having important possibilities in certain parts of the world, especially where rainfall is inadequate, but no longer in favour in England for several reasons. Of these, rising land-values, nuisance from odours, and the danger of contamination of underground sources of water are among the more important.

Turning to the more usual method of disposal, namely discharge into rivers or streams, the treatment required to obviate pollution resolves itself into two fairly distinct parts : (1) the separation and subsequent disposal of the solid constituents of the sewage ; and (2) the oxidation of the dissolved impurities. The continued discharge into a river of the solid matter in sewage, whether organic or inorganic in origin, is harmful, since it is likely to lead to siltation. Moreover, the organic solid matter will decompose and give rise to products having an oxygen demand, as well as being toxic to aquatic life, thus increasing and prolonging the effects of the pollution. The dissolved impurities in sewage, whether arising from the metabolism of the human body or from industrial processes, are, in the main, either chemically unstable or have reducing tendencies, or both. Unless, therefore, they are first stabilized or oxidized their discharge will result in the depletion of the oxygen content of the river-water, with its consequent harmful effects. These effects can be avoided only by first satisfying the oxygen demand of the sewage, thus providing the required stability.

The criteria usually applied to these two main classes of treatment processes are respectively the suspended-solids content and the biochemical oxygen demand (B.O.D.), of the resultant effluent, and the choice of processes to be employed in the design of a treatment works depends primarily upon the standards required for the effluent as measured by these two yardsticks. For the inland rivers of Great Britain the standard recommended by the Royal Commission on Sewage Disposal was 30 parts per million for suspended solids and 20 parts per million for B.O.D., and, for many years past, sewage-treatment works in England have been designed to produce an effluent complying with this standard, a standard which has been found in practice to safeguard adequately the health of rivers without imposing an unduly heavy financial burden on the community. For larger continental rivers, river estuaries and other large bodies of water, lower standards are acceptable in most instances, but tidal conditions must be carefully studied before any definite standard is adopted for discharge into the sea if the amenities of beaches used for bathing and recreational purposes generally are to be preserved. The adoption of standards appreciably higher than the Royal Commission standard, though advocated in some quarters, would undoubtedly prove very expensive, not only because of the high construction costs at present prevailing, but for the more fundamental reason that treatment costs as a whole rise rapidly with only comparatively small improvements in the standard above Royal Commission level.

Treatment Processes Available

Nearly all the treatment processes designed for the protection of natural waters against pollution by sewage derive from Nature's own methods. All organic matter no longer required for any particular life process is decomposed in the layer of soil found on the surface of the earth, and in a different manner in the sea also, and becomes the raw material for other life processes. This decomposition of organic wastes is carried out mainly by micro-organisms of two kinds, aerobic and anaerobic, and it is the harnessing of this bacterial energy upon which many of the sewage-treatment processes now in general use depends. In sewage-treatment practice, these natural processes are concentrated and controlled, those making use of aerobic bacteria being separated from those using anaerobes.

The sewage-treatment processes in general use may be tabulated as follows :—

(1) Dispersal into large bodies of water with or without preliminary treatment.

(2) Irrigation on land, with or without cropping.

(3) Separation of the solid and liquid constituents of sewage,

(*a*) Coarse floating matter by :—
(i) Screens.
(ii) Comminutors (followed by sedimentation).

(*b*) Finer suspended matter by :—
(i) Grit channels.
(ii) Detritors.
(iii) Settlement tanks :—
(A) Plain sedimentation.
(B) Chemical precipitation.
(iv) Filtration.

(4) Stabilization of the dissolved, colloidal and unsettleable constituents by :—
(*a*) Dilution in natural waters.
(*b*) Irrigation on land (with or without cropping).
(*c*) Percolating filters.
(*d*) Activated sludge process :—
(i) Diffused air systems.
(ii) Surface aeration systems.
(*e*) Chlorination.

(5) Sludge treatment :—
(*a*) Digestion
(*b*) Heat treatment } followed by (*c*), or
(*c*) Mechanical dewatering by :—
(i) Drying beds.
(ii) Filter presses.
(iii) Vacuum filters.
(iv) Centrifuges.
(*d*) Composting (with other materials).
(*e*) Heat drying.

(6) Effluent disposal :—
(*a*) Into natural body of water.
(*b*) Irrigation on land (with or without cropping).
(*c*) Reclamation.

(7) Sludge disposal :—
(*a*) On land :—
(i) Distribution in liquid form (for crop growing).
(ii) Trenching.
(iii) Spreading after drying (as manure).
(iv) As filling.
(*b*) Lagooning.
(*c*) Incineration.

The Choice of Treatment Processes

There are a number of factors determining the choice of the processes to be used for the treatment of sewage. These factors are of varying importance but they may be grouped into two principal categories, primary and secondary. In the main, the primary factors influence the general choice of processes, the secondary factors the more detailed, and for clarity of arrangement, this scheme will be followed in the discussion that follows. The primary factors include :—

(1) The standard required for the effluent.
(2) The character of the sewage to be treated.
(3) The size of the system (or quantity of sewage to be treated) both immediate and ultimate.
(4) The ruling economic factors.

The secondary factors include :—

(1) Climatic conditions.
(2) Site conditions.
(3) Materials and labour available for construction, operation, and maintenance.

In general it may be said that the larger the project the greater the choice of treatment processes available unless dilution in a large body of water or irrigation on land is all the treatment required. The converse is also true, the treatment processes found to be satisfactory in practice for very small schemes being extremely few.

Primary Factors

The Standard of Effluent

The standard required for the effluent is, of course, one of the most important factors in the choice of treatment processes, and that standard is determined largely by the size of the body of water receiving the sewage and the subsequent uses made of that water. Where discharge into a very large body of water is possible, treatment of any kind may sometimes be dispensed with altogether, but where coastal and waterside recreational amenities are valued, primary treatment (that is, separation of solid matter) at least is desirable. Gross fouling of bathing beaches may be avoided by maceration of the larger solids, but separation of suspended solids by sedimentation results in an effluent of a much higher quality, and is therefore preferable. The dangers to health through bathing in sewage-contaminated waters are real, and should not be ignored. These dangers are enormously reduced, if not entirely eliminated, by complete treatment (namely, separation of solid matter followed by oxidation), since the reduction in the numbers of micro-organisms causing or associated with disease resulting from such treatment is very great, usually exceeding 90 per cent, and frequently up to 98 per cent.

Where the water of the receiving stream is later used as raw water for public supply, a high standard for sewage effluents is obviously desirable. With this condition in mind, the Royal Commission on Sewage Disposal, 1912, recommended that only where the dilution afforded by the receiving water was greater than 500 : 1, could treatment of any kind be dispensed with. Extensive researches into the problem by American technicians during the early years of the present century indicated that with dilution factors as low as 5 cusecs per 1,000 population (equivalent to 67·5 : 1, at 40 gallons per head per day), the condition of the river remained "unoffensive,"[1] but this criterion is not now considered good enough if the river is subsequently used to provide water for domestic purposes, or even for safeguarding amenities.

Where lower dilution factors applied, the Royal Commission recommended primary treatment, namely, plain sedimentation for dilutions between 300 : 1 and 500 : 1, and chemical precipitation for dilutions between 150 : 1 and 300 : 1. Complete treatment was recommended where the dilution was less than 150 : 1. The Royal Commission standard for sewage effluents, that is, 20 parts per million B.O.D., and 30 parts per million suspended solids, was based upon a dilution factor of 8 : 1, but has since been found satisfactory at lower dilutions.

Assuming that these criteria are sound, and they have never been seriously challenged, it will readily be seen that for most inland communities in England, complete treatment of sewage is necessary, but that in countries where there are large rivers, partial treatment will be sufficient for many of the communities situated near to them.

Apart from irrigation on land, complete treatment comprises separation of solids, usually by sedimentation (primary treatment); reasonably complete oxidation, almost always by biological means, of the dissolved and colloidal impurities, with flocculation of the remaining (unsettleable) solids (secondary treatment); and a final sedimentation stage to remove the flocculated solids and the waste products of the biological activity. Secondary treatment by biological means, whether by percolating filters or by one of the activated-sludge processes, may conveniently be regarded as being carried out in three stages :—

(1) Flocculation (or clarification).
(2) Oxidation of carbonaceous impurities.
(3) Oxidation of nitrogenous impurities (or nitrification).

The purification is carried out roughly in that order, though no very definite dividing line exists between the stages, and much overlapping occurs. The capacity required to achieve purification to each stage depends, of course, on the strength of the sewage, but in general, flocculation may be achieved by low capacities, that is, high rates of application on filters or short aeration periods, for example, 600 to 1,500 gallons per cubic

[1] The references are given on p. 253.

yard per day or 2 to 4 hours aeration at 0·5 to 0·75 cubic foot of air per gallon. The oxidation of nitrogen requires high capacities, that is, rates not exceeding 100 gallons per cubic yard of media per day, and 8 to 9 hours of aeration at 1·2 to 1·4 cubic feet of air per gallon of medium-strength sewage treated. In America, secondary treatment is frequently unnecessary, and where it is adopted purification to the clarification stage is often sufficient, the remaining stages being carried out in the river or other receiving water where the dilution afforded is usually adequate to achieve this with safety. In England, most treatment works have been designed in the past with a sufficient margin of capacity to produce a reasonably well nitrified effluent on their design flows, and have often continued to satisfy Royal Commission standards with a considerable degree of overloading, but if higher standards are to be imposed by the Rivers Boards, as appears possible, secondary treatment to the nitrification stage will be essential, and will have to be followed by final settlement tanks of a design superior to the old-fashioned humus tanks, and equipped with means for the continuous removal of sludge. In addition, it may become necessary to introduce a further stage of treatment for "polishing" the effluent. Final removal of suspended solids by settlement alone is unlikely at all times to produce an effluent having less than 20 parts per million of suspended matter in it, even when the effluent is well nitrified, and higher standards will require additional treatment for the effluent, for example, rapid sand filtration or micro-straining. For such purposes some of the older sewage-works managers used to resort to a device which is very effective, namely, passing the effluent over a small area of grass land. Well purified effluents can readily and consistently be produced having less than 10 parts per million of suspended matter by this means, but it could not be applied at many works except by introducing an additional pumping stage. Slow sand filters may also be used for a similar final "polishing," and in addition they may be made to carry the biological purification to a higher stage by the development of bacterial activity in the filter, but such filters require the expenditure of considerable labour for their effective operation and maintenance.

Stabilization by chemical means, for example, by chlorine, has been used in America but may be ruled out for most countries on the score of cost as compared with biological oxidation.

The Character of the Sewage

The composition of a sewage arriving at the treatment works is determined by:—

(1) The amount and character of the industrial wastes present.
(2) The dietary habits of the contributing population.
(3) The quantity of water used per unit of population.
(4) The type of sewerage system, that is, whether separate or combined.

(5) The length and gradient of the outfall sewer.
(6) The climatic conditions.

Of these factors the quantity and composition of the industrial wastes present in the sewage, the quantity of water used, and dietary habits are the more important factors affecting the choice of treatment processes.

Industrial wastes are now so varied in composition and in their effects on sewage-treatment processes that it is impossible to state, except in very general terms, how they may influence the choice of such processes. Where staple industries exist, the sewage will inevitably contain a preponderance of the wastes from those industries, and the choice of process may thus be determined solely by those wastes. At Bradford, for example, the method of sludge treatment used for many years past was adopted by reason of the high grease-content of the sewage derived from the wool-washing industries of that city. Many industrial wastes are highly toxic and can seriously inhibit or even prevent bacterial action entirely, even when present in the sewage in comparatively small concentrations. Since practically all oxidation processes used in sewage treatment depend upon bacterial action, the content of toxic wastes in sewage must be strictly limited to the proportions which these processes will tolerate. As examples of such proportions it may be quoted that comparatively low concentrations of chrome wastes have been known to put a sewage works completely out of operation, and cyanide wastes have a deleterious effect on the operation of the activated sludge process when present in the sewage treated to the extent of 1 part (as HCN) per million.[2] In the Author's opinion, a concentration of 0·5 per cent of spent ammoniacal liquor from gas works should not be exceeded if the purification processes are not to be seriously affected. In general it may be stated, however, that industrial wastes should be treated at the sewage-treatment works along with the sewage itself, and if the concentrations of wastes are kept below certain limits, and their pH value and temperature controlled, a satisfactory effluent can be produced by the use of the same processes as are normally chosen for treatment of domestic sewage.

Percolating filters are considered preferable to the activated sludge process for the secondary treatment of sewages having a high concentration of industrial wastes, since the latter process is more susceptible to disturbances caused by such wastes. Certain organic wastes, for instance, are liable to cause " bulking " of the sludge. Such difficulties do not have the same effect in filter installations, though " ponding " of filters may be the corresponding effect of some wastes. Alternating double filtration has been found successful in treating high concentrations of milk and other organic washings.[3]

Few trade wastes affect sedimentation processes, though some may improve settlement by coagulation of the sewage solids and colloids. Sludge digestion and especially the yield of gas, may be inhibited by certain

metallic constituents of trade wastes, and others—for example, slaughter-house and meat canning wastes—will obviously impose a greater load on the digestion plant, though a larger yield of gas will probably be obtained from sludges of this type.

In general, difficulties in operation or maintenance of sewage treatment works arising from dangerously high concentrations of industrial wastes could be largely eliminated if all local authorities made full use of their powers under the Public Health (Drainage of Trade Premises) Act, of 1937.

Dietary habits affect the composition of sewage sludge and therefore the capacity and possibly the choice of the sludge-treatment processes adopted. The diet of prosperous communities living in temperate and cold climates tends to include a high proportion of protein and fatty foods, and anaerobic digestion of the sludge produced at the treatment works serving such communities is possible and gives rise to a high yield of sludge gas having a methane content of about 70 per cent. In hotter climates, where such a diet is less suitable, the yield of gas may be insufficient to warrant the cost of the plant required to collect, store, and use it, even if digestion of the sludge is possible.

Where coloured populations predominate, a largely carbo-hydrate diet is probable, and this is likely to affect the method of sludge treatment. Dr Hamlin has reported that the sewage sludge resulting from the carbo-hydrate diet of African communities cannot be digested by normal means, and other methods of disposal have to be adopted at works serving such communities.[4]

In many countries of the East, sand is used for scouring domestic cooking utensils, and the sewage from such communities is likely to contain much more sand and grit than would be expected in western countries. Grit channels and detritors should therefore be designed accordingly, the grit-removal plant in particular being provided with adequate capacity.

In western communities, the quantity of polluting matter in domestic sewage per unit of population varies over a much smaller range than does the water content, the latter varying from about 15 gallons in some country districts in England, to more than 100 gallons (Imperial) in many cities in America, and as much as 250 gallons (Imperial) in the city of Chicago. The strength of the resulting sewages varies accordingly, and the choice of treatment processes may be affected, if not directly then by economic considerations. Thus the cost of constructing sedimentation tanks giving an adequate retention period for sewage from communities using very large quantities of water will be high in comparison with the percentage purification achieved, and other methods of solids removal such as fine screening may be found equally successful in operation, and be more economical. In the secondary-treatment stage, the activated sludge processes are generally considered best suited to weak sewages, and percolating filters, single- or two-stage to be most successful with strong sewages.

A combined system of sewerage will, of course, give a much weaker sewage than usual in wet weather, but there will be much more of it. Treatment works are usually designed to deal with three times the dry-weather flow of sewage, that is, they can accept two volumes of storm-water at the same time as one volume of sewage. Beyond this proportion it is unsafe to go, since although the strength of the sewage is reduced by the dilution the storm-water affords (at any rate after the first flush) the retention times in the sedimentation and biological purification stages will be so reduced as to affect the standard of the effluent. In addition, the hydraulic conditions of the works as a whole will most likely be seriously affected.

For greater quantities of storm-water, separate provision must therefore be made, either in the form of special storm-water tanks at the works, or storm-water overflows, or both; the choice depending largely upon the size of the receiving body of water and the standards required for both dry- and wet-weather effluents. Storm-water tanks may be provided to give settlement only to such quantities of flow as cannot be fully treated in the main works, the settled sewage passing without secondary treatment to the river; or they may act as storage basins, the settled sewage being pumped back for secondary treatment when the flow into the works returns to normal. With a combined system of sewers, this latter condition would be almost impossible to fulfil and would be extremely costly in any event. The former may be achieved without undue expense if combined with overflows on the sewerage system, but not otherwise. The former Ministry of Health recommendations were that storm-water tanks of sufficient capacity to treat flows above three times and up to six times the dry-weather flow should be provided, all flows above the latter figure being diverted to the river without treatment.

It will be readily appreciated that the adoption of such a system will result in considerable pollution of the receiving body of water, especially after a long spell of dry weather. It may be calculated that during a storm of moderate intensity, as much as 80 per cent of the pollution load contributed by the sewage alone may be discharged to the river, and the first flushes are often very foul indeed. It is largely because of considerations of this kind that separate systems of sewerage are now preferred.

A long outfall sewer with a flat gradient will often delay the arrival of the sewage sufficiently to produce septicity. This increases the load on the purification plant and may make settlement of solid matter more difficult. Septic sewage is invariably more difficult to treat than fresh sewage, and although the choice of processes may not be affected, the capacity of the secondary-treatment process will have to be greater if this type of sewage is received.

Climate has several easily recognizable effects, direct and indirect, on the composition of sewage. In hot climates, for example, the general tendency will be for sewage to be weaker by reason of the greater quantities

of water used, but it will also tend to become septic more quickly and solid matter will be disentegrated, making separation by settlement more difficult. The effect of climate on the choice of sewage-treatment processes will be discussed later.

The Size of the Scheme

Assuming that complete treatment is necessary, the quantity of sewage to be treated should not, in theory, affect the choice of treatment processes. In other words, small works could be designed to make use of the same processes as large works, and operated just as successfully and efficiently. In practice, this is not so, since the efficient operation of a sewage-treatment works making use of the most suitable processes for the type of sewage to be treated demands not only constant attention but also the application of a wide variety of skill, knowledge, and experience. The smallest works designed to treat the sewage from single dwellings, groups of a few dwellings, or small institutions, have in practice, and of necessity, to operate with the minimum of intermittent attention, usually of an unskilled type. The choice of process is therefore limited to those which can operate reasonably well under these conditions, that is to say, the septic tank or the two-storey tank for the separation and digestion of solid matter, followed by the percolating filter or subsurface land irrigation. The upper limit of population for small schemes of this type has recently been recommended as 300.[5]

Above this limit, the size of the scheme is in general reflected in the degree of mechanization introduced, and thus in the complexity of the processes used. Simplicity should never be sacrificed to complexity unless increased efficiency or greater economy results, and the more complex and highly mechanized processes, such as the activated sludge process of biological oxidation, and mechanical dewatering and heat drying or incineration of sludge, should therefore be reserved for the larger undertakings. All these processes require the constant attention of fully qualified and experienced chemists for successful operation and of mechanical and electrical engineers and skilled mechanics for maintenance. The simpler and less mechanized processes for both purification and sludge disposal, that is to say, percolating filters and air drying respectively, are therefore more appropriate for medium-size schemes. In the Author's opinion, the lower limit of population for schemes using the activated sludge process is about 50,000.

The Ruling Economic Factors

Although essential to health, sewage treatment and disposal has never been a profit-making undertaking and its cost must be borne by the community as a whole. The capital cost and the maintenance and operating costs must therefore always be paramount considerations in the choice of treatment processes even in the most prosperous communities.

Up to the present no artificial process has been found to compete, either in cost or in efficiency, with the natural processes making use of bacterial energy. In countries such as India, where extreme poverty is the rule rather than the exception, there are many thousands of communities still unable to afford either a piped water-supply or a sewerage system at all, but assuming the minimum standard of prosperity (together with the desire for a reasonable standard of sanitation—a desire not yet in evidence amongst all peoples), the least costly forms of sewage treatment, both in capital expenditure and running costs are, of course, dispersal into water or irrigation on land. As the prosperity of a community increases, so land values increase, and the tendency is therefore towards the concentration of sewage treatment, that is to say, the choice of processes requiring the minimum of land. Saving in cost of land by concentration is, however, offset to a considerable extent by increase in capital cost and usually in running costs also.

Assuming that dispersal in water is not possible, and this is true of most communities, purification by irrigation requires the greatest area, and that by the activated sludge process the least; treatment on percolating filters being intermediate. The ratios are approximately as follows :—

Activated sludge—1.
Percolating filters—10.
Irrigation on land—800 to 1,200.

It will be seen that concentration results in complexity and mechanization of processes for purification, and the same can be said of the sludge-disposal processes. The greatest area is required for distribution on land, trenching, and lagooning, the least for the mechanical dewatering, heat drying, and incineration processes, drying on underdrained beds being intermediate.

The desire for improved amenities tends to increase with increasing prosperity, and it is fortunate, but entirely fortuitous, that reduction in the nuisance from unpleasant odours, flies, and mosquitoes follows as a result of the concentration of purification processes from land treatment, through percolating filters to the activated sludge process, and also of sludge treatment from spreading on land or lagooning, through drying beds to mechanical dewatering and heat drying.

The tendency towards increase in costs through concentration and mechanization may be offset by the use of sludge gas for power production and by the sale of dried sludge (in a suitable form) as a manure or soil conditioner. Sewage farming as such, that is to say, growing crops on land irrigated by sewage, though largely abandoned in England and America, mainly as a result of increases in land values, may be chosen with advantage in other parts of the world, especially in poorer countries and in arid or semi-arid climates, but the health hazards associated with the direct consumption of crops so grown should be considered, because they are

undoubtedly serious. Indirect systems, such as the growing of grass or other green crops for fattening cattle, as practised for example in Melbourne, Australia, and in parts of South Africa, are much to be preferred.

Secondary Factors

Climatic Conditions

As previously mentioned, climatic conditions affect the character of sewage both directly and, by its influence on dietary habits, indirectly. The effect of climate on the choice of treatment processes is therefore partly through its influence on the character of the sewage, but it may also have more direct effects. In very cold climates, irrigation of either sewage or purified effluent on land is impossible for a large part of the year, and it is obvious that this method of disposal is more effective where rainfall is low.

Processes resulting in the separation of the solid matters in sewage are not generally affected by low temperatures until freezing occurs, but high temperatures affect separation by sedimentation in that the time of retention in the tanks increases the septicity of the sewage, and the sludge produced rapidly goes septic also and may rise to the surface. Sedimentation tanks for hot climates should therefore be designed with as short a retention period as possible, and preferably with continuous sludge removal.

In cold climates it may be necessary to enclose percolating filters to prevent freezing, whilst in regions where high temperatures prevail, ponding may result from the increased bacterial activity. Similarly, it has been shown that activated-sludge plants increase in efficiency with rising temperature, owing to increased bacterial activity,[6] but this may be offset to some degree by the lower saturation level of oxygen in water at higher temperatures, a fact which definitely affects the pollution load a river can tolerate at different seasons of the year, or in different climates.

In arid countries water may be so scarce, or so difficult and expensive to obtain, that sewage effluent has to be reclaimed either for industrial use or for the irrigation of crops. Complete treatment would be essential in such cases, possibly followed by a final "polishing" process such as sand filtration or micro-straining.

Many of the older methods of sludge disposal are seriously affected by climatic extremes. Application of liquid sludge on land either on the surface or by trenching would be extremely difficult or even impossible for long periods in cold climates, and the same may be said of drying on beds. Considerable storage capacity, either in tanks or in the drying beds themselves, is therefore required to allow for these periods of inactivity, the actual capacity depending upon the maximum period of such inactivity likely to be experienced. Sludge-digestion tanks in cold climates should be heated and may require insulating against excessive heat losses. Composting techniques are likely to be most successful in hot climates, and may be found quite unworkable in cold.

Rainfall has less effect on sludge-disposal processes than has temperature, largely because the regions of highest rainfall are in tropical countries, where evaporation is correspondingly greater.

Mechanical dewatering and heat-drying methods of sludge treatment are not directly affected by climatic conditions so far as is known, and are therefore preferable, other factors permitting, where older processes are seriously affected by weather conditions or climatic extremes.

Site Conditions

Both surface and subsurface conditions on the site chosen for the sewage-treatment works may be major factors affecting the choice of a number of treatment processes, and will certainly affect the design of tanks, channels, etc.

Of the surface conditions, slopes and contours are of great importance. This is obvious where irrigation is practised, and it may also determine the choice between percolating filters and one of the methods of activated-sludge treatment, the latter requiring little more than as many inches as the former requires feet of head-loss for operation.

Subsoil conditions affect structural design more than actual choice of treatment processes but it is again obvious that irrigation on land will be more successful, and require less land, if the subsoil is light and sandy, with a water table some distance below ground level. Where the water table is near to ground level, or the site liable to flooding, irrigation is unsuitable and would not normally be chosen. Similar subsoil and surface conditions may determine the general operating level of the treatment works and thus the static pumping head, or even whether pumping will be necessary or not; the great increase in capital cost of construction in waterlogged ground more than outweighs the additional pumping cost, although this is continuous. Thus in a large regional scheme, with the design of which the Author was concerned, such subsoil conditions made it necessary to build the treatment works largely above ground and thus to increase the pumping head by several feet in order to avoid extremely heavy construction costs.[7]

Where piling is necessary, tank designs requiring the minimum area in plan should be chosen, and in such cases the choice of secondary treatment, if required, would be the activated sludge process rather than percolating filters, other factors permitting, by reason of the greatly reduced area required.

The proximity of the site to dwellings and the consequent desirability of preserving amenities by avoiding nuisance from odours or flies may determine whether certain processes are chosen. The farther the sewage works is from the town the greater is the length of the outfall sewer, and it is therefore uneconomical to site the works very far away, even if natural fall is available. Even if the site is remote from dwellings at first, growth and expansion of the community served may ultimately result in houses

or factories being built up to, or even all round the treatment works. The question of amenities should therefore always be considered.

Odours arising from screening processes may be confined by housing such processes in buildings, but can be avoided altogether by using comminutors. Most modern designs of sedimentation tank, if operated properly, do not give rise to odours on any appreciable scale, but frequent or continuous removal of sludge is necessary and the tanks must continue in operation during sludge removal, therefore, they must be equipped with mechanical scrapers, if this comparative freedom from odours is to be achieved at all times.

Of the secondary treatment processes, irrigation on land often gives rise to serious odour nuisance, and percolating filters are not always blameless in this respect. Filters have the added disadvantage of attracting a wide variety of insect life, which in fact plays an essential part in their successful operation. Some of these insects can become a nuisance in nearby dwellings, and although themselves harmless, might assist in spreading disease under certain circumstances. The only way of ensuring complete freedom from both fly- and odour-nuisance in secondary treatment of sewage is to adopt one of the activated sludge processes.

It is a truism that the solution of one problem often creates another, and in no field of technology is this more true than in sewage-treatment practice. Surplus activated sludge, if not disposed of rapidly, can give rise to odours far more offensive than those arising from any normal percolating filter installation. Primary sludge is also offensive, but if it is first digested, it may be spread on land or on drying beds without causing any serious odour problem. A mixture of primary and surplus activated sludges can also be made reasonably innocuous by digestion, so that this process is essential for the treatment of both primary and secondary sludges, if odour nuisance from the subsequent drying on open beds is to be avoided. Vacuum filtration of activated sludge, provided the sludge is fresh, that is to say, before decomposition has set in, can be accomplished entirely without odour nuisance, but unless the moisture in the filter cake is rapidly removed (for example, by " flash drying "), decomposition, with the inevitable odours, may result.

Materials and Labour Available for Construction, Maintenance, and Operation

Where partial or full treatment is necessary, the materials and labour available for the construction, maintenance, and operation of the treatment works may determine not only the design of various parts of the works but also, in some circumstances, the choice of treatment processes. This applies mainly to the less highly developed regions of the world, and not to Great Britain, parts of Europe, or the United States, where most or all the materials and skilled labour required are readily available. In more remote areas where skilled labour for maintenance and operation may be

impossible to obtain, both treatment processes and design of tanks, channels, etc., should be of the simplest, mechanization being avoided as much as possible, and local materials of construction used. Land treatment would be the first choice in most cases, but if this were not possible or desirable, settlement in simple earth tanks, followed by purification on percolating filters should be adequate. Sludge disposal should be by trenching, lagooning, or drying on open beds. Such complex and highly mechanized processes as the activated sludge systems, or mechanical dewatering and heat treatment of sludge would not only be quite uneconomical, but would soon fail through lack of skilled labour and adequate resources for maintenance. For similar reasons, processes making use of imported chemicals or fuels would be out of place and uneconomic, and should therefore be avoided. In brief, the less highly developed the area served, the less likely are skilled craftsmen and technicians to be available for maintenance and operation of sewage-treatment works, and therefore the simpler should be the processes chosen, so that the works may be successfully operated and maintained by comparatively unskilled labour.

Conclusion

In attempting to outline and discuss briefly the more important factors in the choice of sewage-treatment processes, the Author has deliberately ignored personal preference. This used to be of greater importance than it is today, so much so that some of the older sewage works in Great Britain reveal the identity of their designer by his personal preferences, just as the works of an architect or composer are recognizable by their style. In these days, standardization of design, mass production, and specialization militate against individuality in the design of engineering projects, but it is never likely to be eliminated entirely from the field of sewage treatment. It has often been said that no two sewages are alike, and when the other factors mentioned in the foregoing pages are added, it will be readily appreciated that there could never be exact similarity between sewage-treatment projects of any size. Individuality should therefore lie in the particular combination of material factors determining the choice of treatment processes rather than in personal preferences. The skill of the designer lies in the selection of that combination of processes best suited to produce the effluent required in the most economical way. He should restrict his personal preferences to the choice of those proprietary items of equipment, of which there are now available so many excellent and well tried examples, for supplementing or assisting the main processes of treatment.

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Discussion

The Author, when introducing the Paper, emphasized the wider aspects of public health which it was the purpose of sewage treatment to preserve, namely, the health of rivers and consequently the amenities associated with them, and especially the purity of the water of those rivers when it was required for domestic and industrial use. In the past, the greatest destroyers of the amenities associated with rivers, lakes, and seashores had undoubtedly been sewage and industrial wastes, through their discharge without sufficient treatment beforehand to make their toxic and deoxygenating properties ineffective.

For many years past, the public conscience had been slowly but surely awakening to the condition of the rivers of Great Britain, and had now culminated in legislation which gave adequate powers to public authorities for the prevention of river pollution, namely, the Public Health (Drainage of Trade Premises) Act, 1937, and the Rivers (Prevention of Pollution) Act, 1951. Under the powers vested in local authorities by the former Act, the discharge of toxic industrial wastes into the public sewers could be controlled or even prohibited, and the damage or disturbance to sewage-treatment processes that such wastes were liable to cause could thus be prevented or at least minimized. Sewerage authorities were consequently not prevented, through that cause at any rate, from fulfilling their legal obligations to the river authorities to discharge an effluent that would not injure the river in any way, for under the latter Act, such an obligation was in fact imposed on all authorities of that kind.

The Author also referred to the changing conditions of labour and employment which had led to a tendency towards mechanization of most sewage-treatment processes. That ranged from such improvisations as

the use of miniature bulldozers for removing sludge from flat-bottomed sedimentation tanks, or various mechanical devices for lifting sludge from drying-beds, to such processes as vacuum filtration and heat-drying of sludge. The extent to which mechanization could be economically applied to sewage-treatment works, whether existing or under design, was always debatable. Much had been done in Britain in the past towards achieving a degree of mechanization by the ingenuity of sewage-works managers having a mechanical bent, but there was no doubt that mechanization of old works could never be much better than improvisation, and that its full advantage could only be realized when applied in the design stages.

Furthermore it should not be forgotten that all mechanical plant required maintenance, and some of it required continuous skilled or semi-skilled attention for its successful operation. It could not be assumed, therefore, that its application to all sewage-treatment processes at works of all sizes would inevitably result in economy. Its indiscriminate use might well have the opposite effect, but on schemes of any magnitude, and even of moderate size, extensive mechanization had undoubted advantages, especially if power was generated from sludge gas. Mechanization could only have limited application at small works, except under some system of regional control with mobile maintenance and a central workshop for carrying out repair work, etc. Such a system would be quite feasible for rural districts having a number of small sewage works, and in that event such works could well make use of more mechanical plant than had been usual in the past.

Mr C. B. Townend said that a review of the kind given in the Paper emphasized the great extent of the progress made in the treatment of sewage in the past 50 years or so. At the beginning of the century, the main problem had been to find some satisfactory way of treating sewage at all, but the point had now been reached when one was almost bewildered by the wide choice of so many methods and combinations of methods which were available today. The Author had done a great service in clarifying the various factors which had to be involved in making any such choice.

Two of the main factors which the Author had mentioned were the degree of treatment required and the volumes of sewage which should be treated in wet weather. In both cases a certain practice had grown up which had been based on reasonably sound principles and had worked quite well in general during the past period of evaluation. Nevertheless, it could only be described as a rule-of-thumb procedure, and one could not help feeling that a new era was beginning, in which a more flexible practice would gradually emerge.

So far as effluent standards were concerned, that change was foreshadowed by the Rivers (Prevention of Pollution) Act, 1951, to which the Author had referred in his opening remarks. Under that Act, the River Boards would no doubt fix standards, which had more relation to the

circumstances of any particular river. The Royal Commission standard, to which the Author had referred, had proved to be a very good one in practice for the average river where adequate dilution had been available; but future standards would have to take into account more and more the self-purification capacity of the river in each individual case. As the Author had said, in many cases the Royal Commission standard was very conservative and might be unduly stringent, whilst in other cases it might not be good enough. In his introductory remarks, the Author had rather predicted that the standards would go up and not down, but that, of course, remained to be seen.

On the question of wet-weather flow, the true criterion was the strength of the liquid which ultimately reached the river. Hitherto, it had been the practice to give full treatment to three times the dry-weather flow and partial treatment by sedimentation to a further volume of three times the dry-weather flow, up to a total of six times the dry-weather flow; but that procedure would undoubtedly produce a great range of variation in the effect on the river itself. Similar towns would have very much the same impurity load per head of the population, although their water consumption might be very different. In dry weather, one town might have a relatively small volume of strong sewage, while another might have a larger volume of weaker sewage. In wet weather the same strength of sewage should be reached when the total gallonage of diluting water reached the same point in terms per head of population in both cases. Therefore, the practice of giving full treatment or partial treatment should be rather based on the gallonage per head without regard to the dry-weather flow. In fixing such a gallonage for an industrial town, possibly an allowance would have to be made on some population equivalent basis to cover the trade effluents of that particular town, as was common practice in America.

A change in policy on those two particular points towards a greater flexibility would undoubtedly have some considerable bearing on the design of the works concerned.

The Author had quite rightly referred in several places in the Paper to the increasing complexity and mechanization of modern processes, but he seemed to infer at the same time that modern processes were becoming more expensive. But that did not follow at all. Taking mechanization first, in Great Britain the main decisions had up to now practically always been taken on questions of cost. Mr Townend thought that most towns considered cost absolutely first, and surely mechanization was usually adopted to save manpower and cost. After all, manpower was today a very expensive commodity, and it had to be reduced to the minimum wherever possible.

It was true that cases did arise where mechanization was adopted for its own sake, in order to improve amenities or working conditions where labour was no longer available for doing the more unpleasant manual

jobs; but even so, in nearly all cases, there would be a saving in some way or another.

So far as modern processes were concerned, it had to be remembered that they had all been originally introduced in competition with earlier procedures, and one had to be very careful in comparing the cost of a modern plant constructed today with the cost of an old-fashioned plant which might have been constructed 40 or 50 years ago. When originally introduced, the percolating filter, in spite of what might be thought to the contrary, had been fully competitive with land treatment, and the same applied to activated sludge with regard to the percolating filter. The activated-sludge process was considerably cheaper in capital cost than the percolating filter, although it involved heavier power consumption. That had been regarded in the old days as a point in favour of the activated sludge, since with rapid improvement of processes it had been good policy to reduce capital expenditure—money locked away in capital assets which might quickly become obsolete. Today, in the case of the national power-supply undertakings, that policy was being reversed by sinking more money in hydro-electric schemes with a view to conserving Britain's coal resources. With activated sludge, however, the consumption of power did not need to be regarded from that angle, for, by a stroke of good fortune, the activated-sludge process had developed almost simultaneously with sludge digestion, and the use of methane for power production had created a situation in which, so long as the sewage arrived at the works, it would bring its own source of power indefinitely. That was a fact that should appeal to the sense of the fitness of things in the mind of every engineer.

Another point regarding the use of activated sludge, on which the Author had commented, concerned its ability to deal with stronger sewage and trade waste. Mr Townend thought it ought to be pointed out that activated sludge had been employed for very many years for the strong industrial sewages in Manchester, Sheffield, Birmingham, and many other places in the industrial North. At one well-known works it was claimed that the results with activated sludge were actually superior to those with the percolating filter working in parallel on the same difficult sewage.

There could be no doubt today that where the activated-sludge process was properly established in a well-designed and well-operated plant, the process would handle adverse sewages with considerable robustness. It was always possible in the case of a strong sewage, to reduce its strength by re-circulation—a technique which was, of course, equally being employed to improve the performance of the percolating filter.

Mr C. D. C. Braine noted that in the Paper the size of system and economy were listed amongst the four primary factors which would determine the Author's choice of sewage-treatment process; Mr Braine suggested, however, that those two particular factors were so closely related that they could hardly be regarded separately, and he showed two lantern slides illustrating that point.

The first slide was a graph showing how rapidly unit costs per million gallons of dry-weather flow on small plants decreased with increased size of plant until one reached plants about the size of the Maple Lodge Works. The curve then flattened out and became asymptotic when plants the size of that at Mogden were considered, after which there was apparently little to be gained by increasing size.

It was interesting that the actual curve shown on the slide had nothing to do with British plants, but was based on Velz's [1] curve of plant costs in the United States, brought up to date at current rates of exchange. It was also, however, almost the "best fit" curve for typical activated-sludge-plant costs in Great Britain, which was a coincidence. The economic size of any given treatment plant could hardly be divorced from the consideration of the sewer serving it, and the second slide compared the unit costs of discharging the dry-weather flow of sewage through different sizes of sewer on the assumption that they were designed to carry a maximum of six times the dry-weather flow and that the velocity in each case was $3\frac{1}{4}$ feet per second.

The curve showed the relatively high unit cost associated with small sewers as compared with the low unit costs of large sewers. Little was gained, apparently, by using sewers of more than about 10 feet diameter. The curve shown needed qualifying because where the sewage had to be pumped, the most economic size of sewer could be determined mathematically. For instance, using Manning's formula and assuming a roughness value for a sewer, say, 10 years old, of $n = 0{\cdot}014$, it could be shown that for a low-level sewer constructed in tunnel through uniform ground the most economic size of sewer was given by the expression :

$$D = \frac{19}{3}\sqrt{\frac{0{\cdot}000285Q^3K_3}{K_1}}$$

where Q denoted maximum discharge ($6 \times$ dry-weather flow) in cusecs ;
D denoted diameter of sewer in feet ;
K_1D denoted loan charge in £ per linear yard of sewer (for small differences in diameter) ;
and K_3 denoted inclusive cost in £ of pumping per water horse-power.

From that it appeared that it was normally cheapest to tilt a sewer and so reduce its diameter, notwithstanding that the head on the pumps would be increased.

The two graphs emphasized not only that enormous economies were to be gained by centralizing sewage works, but that even if that could not be done, astonishing economies were often to be had merely by linking together nearby works ; and broadly, since the mixing of different kinds

[1] C. J. Velz, "How Much Should Sewage-Treatment Cost ?" *Engng News-Rec.*, vol. 141, No. 16, p. 84 (14 Oct., 1948).

of sewages resulted in a mixture that was easier to treat than some of the individual sewages, those amalgamations, small or large, might considerably affect the choice of process.

Those processes were subject to constant change and big changes were occurring, for engineers were now being compelled to take account not only of the acute shortage of labour that existed at most works, but also of the (perhaps allied) social stigma that nowadays attached to the performance of dirty work. Labour was so scarce at present that on many old works with hand-cleaned rectangular humus tanks, the intervals between cleanings were getting longer and longer, with adverse effect on the effluents from those tanks. Similarly, because it had been generally considered that mechanical equipment on storm tanks would lie idle for such long periods that mechanization was uneconomical, many storm tanks, in the absence of mechanization or of sufficient labour, stood full or partly full of storm-water and sludge for months on end, and at the onset of a storm that ghastly malodorous material was discharged into the nearest stream, by which time it was often so septic that it bore no resemblance to fresh storm-water discharged from a clean tank.

The provision of storm-water tanks of large capacity by their forebears had been prudent and proper in those days because labour had existed for the operation of those tanks, but in many cases that no longer applied, with the result that, judging by Mr Braine's own experience, many of those large tanks no longer prevented pollution but, instead, had become an actual and active source of pollution. That seemed to apply particularly to small and medium-sized works.

In each of the above cases a change of process was desirable, and it seemed to him that it would be possible to kill the two birds with one stone if filter effluent and storm-water were settled in the same tank. In effect, then, instead of the common arrangement of, say, three rectangular humus tanks and three similar but rather larger storm tanks, there would be two or three relatively large circular combined tanks capable of settling properly the whole of the flow that would have been handled by the six smaller tanks. Each of the combined tanks would be equipped with sludge-scraping mechanism so that both storm and humus sludge could be continuously evacuated from them.

Such tanks would be far larger than normal humus tanks, so that in dry weather they would give better settlement with less danger than usual from rising sludge because of the scrapers, whilst at the onset of a storm, tanks which would always be full of filter effluent could have their whole contents slowly displaced by a mixture of filter effluent and storm-water. Thus, the oxidation of the storm-water would actually commence before any of it had reached the river. Moreover, since succeeding waves in a rising stream overtook the earlier ones, the flush of mixed storm-water and filter effluent would overtake the original filter effluent on its way downstream, so that not far below the outfall the proportion of filter effluent to

storm-water in the stream would actually increase, to the benefit of the river.

From the point of view of the health of the rivers, that surely represented a great improvement on what was happening in so many places today. It also had the virtue that it permitted a notable economy both in constructional and in operational costs. It was perhaps worth recording that the idea had already had the blessing of one of the principal drainage authorities and of an important Rivers Board. It might well prove that yet another new technique had been evolved.

The rapidity with which sludge could now be evacuated from tanks owing to improvements in sludge scraping equipment [1] might make it possible to use larger circular tanks than any hitherto used in Great Britain.

Mr J. T. Calvert commented on the Author's reference to the areas occupied by the different oxidation processes. The Author had given the ratio of those areas as: activated sludge—1; percolating filters—10; and irrigation on land—800 to 1,200. There was, of course, no doubt about those figures, but Mr Calvert felt that they tended to give a somewhat misleading impression in that the area of a sewage works occupied by the aeration process was perhaps one-quarter or one-half of the total area. In the case of a works with conventional sludge-drying beds, the beds practically doubled the area compared with filters; and, in addition, it was not possible to cover the area completely since it was necessary to have space for storage, etc., which probably again doubled the area of the works. He felt that the difference between activated sludge and percolating filters probably reduced the total area only by about 20 to 25 per cent.

The second point that he raised concerned secondary factors. The Author had stated on p. 249 that in arid countries, where the sewage was used for the irrigation of crops, complete treatment would be essential in such cases, possibly followed by a final polishing process, such as sand filtration or micro-straining. Mr Calvert's view was that where the effluent was being used for irrigation, the need for a final polishing was non-existent. In such cases the effluent was being used not only for its water content but also for its fertilizer value, and the presence of a certain amount of humus was more likely to be beneficial than detrimental.

He was rather sorry that the Author had said that in present times, with standardization, mass production and specialization militated against individuality in design. Mr Calvert felt that there was still scope for sound engineering and economy by good engineering, which was really the characteristic of the individuality of design.

The Author had given a figure of 50,000 as the minimum population for which activated sludge should be used, and he wondered whether the Author had any views as to a similar figure for the minimum population

[1] C. D. C. Braine, "Some economic aspects of Sewage Works Design." Ann. Conf. Inst. Sewage Pur., 1952.

where vacuum filtration might also be used. The Author was running one of the few plants in Great Britain where there was vacuum filtration, and it was to be hoped that in due course there would be a Paper on the subject. Had the Author yet formed any firm views as to its economy ?

Mr W. T. Lockett observed that the Author had stated that treatment works were usually designed to treat three times the dry-weather flow of sewage. That was true. Nevertheless, in view of the considerable differences in water usage in different towns and communities and the steady increase in water consumption by the population generally, there would appear to be some need for mentioning in a Paper of the kind under discussion that some experts, notably Mr C. B. Townend, were of the opinion that that basis of design was rapidly becoming out of date. A more reasonable basis would appear to be full treatment of all flows of sewage up to a rate of 120 gallons per head of the population per day and partial treatment in storm-water tanks for flows in excess of that, up to a rate of at least 240 gallons per head of population per day.

The Institute of Sewage Purification, in its Memorandum in 1945 on the establishment of new towns in Great Britain, had put forward similar recommendations, as shown by the following extract [1] :—

> "The ideal practice would be to ensure that no liquid which has once entered a soil sewer should be discharged to a stream without some form of treatment, and the only practicable way of achieving this ideal is the adoption of a substantially separate sewerage system designed to include polluted surface waters from yards, etc., but provided in such a manner that the maximum flow will not exceed the capacity of the sewer ; this capacity should be allowed at the rate of at least 240 gallons per head per day. Provision should be made in the sewage purification plant for full treatment to be given to all flows up to a rate of 120 gallons per head of population per day for domestic sewage plus three times the average flow of trade effluents, together with partial treatment for all flows in excess of this rate arriving at the sewage works."

Mr Lockett said that he did not wish to discuss at length the phenomenon of bulking of activated sludge, but pointed out, with regard to the Author's observation that certain organic wastes were liable to cause bulking of the sludge, that some enlargement of the volume of activated sludge on addition of strong sewage or strong organic waste was largely a natural phenomenon and as the aeration proceeded there was a contraction in the volume. Moreover, difficulties that had arisen in the operation of activated sludge plants on account of bulking in the past had often arisen from the neglect of some of the fundamental requirements of the process, such as an adequate supply of oxygen and effective mixing of the sludge and sewage, and from failure to provide a sufficiently long aeration period or retention period.

[1] J. Inst. Sew. Purif., 1945, Part 2, p. 190.

The Author had not mentioned a factor of prime importance in connexion with sewage treatment at the present time, namely, the introduction of synthetic detergents. Such effects as foaming on sewage-treatment plants and inability of the biological sewage-treatment processes to eliminate or oxidize some of the compounds, which resulted in the discharge of the detergent substances (along with the works effluents) to rivers, where also they gave rise to foaming and possibly other troubles, were fairly well known. Mr Lockett said he did not intend to stress those points, but would say that, however optimistically one approached a subject of that kind, as time went on one became more conscious of the possibility of the harm that some of those compounds might do.

As already mentioned, some of them passed from the sewage-treatment works into the rivers. As he viewed the matter, however, it did not appear that they passed away simply as a solution of detergent in water, such as, for example, common salt or calcium nitrate ; they passed away into the rivers in combination with, or attached to, molecules of certain semi-colloidal sewage matters, principally degradation products and readily decomposable substances extracted from the sludge or from the filter. Thus there seemed to be a fair possibility that those detergents, with their accompanying sewage matters, would tend to accentuate any tendency for decomposition which might arise in a river.

That was little more than an hypothesis at the moment. Certain observations in actual practice had led to the formulation of the hypothesis, and experiments were in progress to substantiate it or otherwise. Nevertheless, it was well conceivable that, because of the presence of detergents in sewage effluents, oxidation of sewage to the stage of appreciable nitrification would become more necessary than in the past, and fairly well clarified effluents containing little or no nitrate would not be quite good enough for discharge into rivers which were not well oxygenated; and that storm-water and sedimentation sewage liquors passing into rivers and estuaries, even at the present time, were more liable to give rise to putrefaction and nuisance than they would have been had the public been still content to use ordinary soap and soda for laundry and kitchen purposes.

Mr H. D. Manning commented first on a small point which arose on p. 241, where the Author had stated : " Where discharge into a very large body of water is possible, treatment of any kind may sometimes be dispensed with altogether. . . ." He had himself been brought up on the idea that disposal by dilution was a form of treatment, a form which the Author had omitted from his list, the implication being that disposal by dilution was disposal rather than treatment.

Several speakers had referred to standards, and Mr Manning said that he wished to refer to the very serious difficulty in which designers found themselves today. The 1951 Act had, of course, given River Boards the power, if not the duty, to lay down standards, but since the passing of

that Act he had noticed that a number of well-informed speakers on the subject had made it very clear that it was going to be many years before anything came of that question of fixing standards. One heard of working parties and of investigations by a number of people, but nobody held out much hope of a new series of standards. Those standards would not necessarily be higher; indeed, he would expect many to be lower (if one could use the terms "higher" and "lower") and some to be higher than the Royal Commission standards. But the main theme of the experts looking into the matter was that the standards should be different in character, and that was very disturbing to the designer, who really did not know very much about the chemistry of the thing and was simply trying to build sewage works to do the job.

The designer had another complication in the case of the new towns, in that he did not know what he was going to deal with or how much of it there was going to be. In the case of a certain township in the Lea Valley, Mr Manning could remember arguments going on for months before any information or guidance could be obtained as to the quantity of water that was to be provided for in that town and what sort of quantity of sewage was to be expected. It had been a completely new problem, so that the designer had been in the position of having to design treatment works without knowing what he had to treat or what standard of effluent would be required.

The Author relegated site conditions to the status of a secondary consideration. From one point of view that might be quite right, but Mr Manning quoted one case where subsoil conditions had had a very direct bearing, as it happened, on the choice of treatment process in the broadest sense.

In a town of about 50,000 population, the only site that could be found for the treatment works which were necessary was $2\frac{1}{2}$ acres in extent and quite incapable of being extended. The site was surrounded by buildings and ranged in depth from 0 to 30 feet of silt overlying rock. He thought it was fairly obvious that in that case one thought in terms of activated sludge rather than filters, because he did not think it would have been possible to get the filters on to that site. The interesting point was that on the particular spot where the aeration plant needed to go, it had been necessary to go down about 20 feet to the rock before getting a foundation of any kind. That had proved to be the determining factor leading to the adoption of mechanical surface aeration, owing to the depth of the tank needed on that particular site.

One other point which had been touched upon already in the discussion was that of labour and operation of works. He felt very strongly indeed that it would be wrong to cling to the idea that simplicity was essential in treatment works because it was difficult to get people with sufficient sense to operate the works if they were made at all complicated. That had been true in the past and was still true to some extent, but he

thought that the situation was changing very rapidly and that the time might well be coming, and in certain areas had come already, when the problem was to get people to do the rough jobs. One could get people to do the fairly intelligent jobs. Whilst designs should by all means be kept simple, he thought that the idea should be abandoned that it was necessary to design foolproof works on the assumption that nobody would ever go near them or do anything to them at all in the course of the year, except cut the grass!

Mr I. C. Forbes said that the Paper had served to emphasize the fact that the advances in sewage disposal during the past 25 years had been so great that no local authority or engineer could afford to ignore them. The controlling factor at present was the money factor, and there were many schemes now in abeyance owing to what was euphemistically called by the Ministry of Housing and Local Government, "incorporation in the national investment programme." At present, authorities in the rural areas were forced to envisage cheap and inferior engineering standards where the general question was not "Will the system be obsolete in about 70 years' time?" but "Will it still be a system before the loan period expires?"

That brought them to the point of slashing the capital cost of works. Beginning with sedimentation tanks, the transporting capacity of water on particles of unit size and mass varied as the sixth power of the velocity; so that if the transporting capacity was given as:

$$\alpha = V^6$$

then with a velocity $V = 1$, the transporting capacity α would also be 1. With a velocity $V = 3$, the transporting capacity would be 729. It would thus be seen that any slight drop in velocity would considerably lessen the transporting capacity.

Following on that hypothesis, in an experiment on sedimentation tanks which he had done several years ago, it had been found that the retention period of the tank was 20 minutes. The inlet penstock had been screwed down to allow only sufficient sewage through-flow to make the retention period 15 hours, and subsequently it had been found on the "oxygen absorbed" and the "Gooch crucible" tests that the tank effluent was not materially improved by the extension to the 15-hour period.

On the subject of filter beds, Mr Forbes mentioned that he had for a number of years worked under the jurisdiction of the late L. F. Mountfort, A.M.I.C.E., who had conducted experiments into the purification of liquor on filter beds at varying depths. In that particular group of experiments it had been found that 68 per cent of the purification was achieved in the top 18 inches of the medium, so that if there were two filters of 18 inches depth placed in series, there would be some saving in cost over that of a deep filter. It would also appear that the maximum loading capacity of filters had not yet been reached, since in another experiment

which he himself had conducted on two 100-foot-diameter filters, 6 feet deep, a dosage rate of 310 gallons per cubic yard per day had given 50 per cent purification, and had reduced the oxygen absorption figure of the effluent liquor from 6·8 parts per 100,000 to 3·2.

Living in an agricultural district and seeing continuous efforts for the reclamation of marginal lands, he was greatly impressed by the need for the conservation of land for food production, and he felt certain that there should be a return to mechanical sludge filters, with a view to conservation of the land used for sludge-drying areas, and getting it back into allotment gardens and smallholdings.

With regard to the fouling of beaches, and spoliation of amenities, he was afraid he could not share the Author's syncretism on the subject. Mr Forbes did not think it was extensive, and had always been under the impression that salt water was a very strong bactericidal substance, and he did not think that any pathogenic bacteria could live in seaside water; but perhaps they might be able to get some expert opinion on that from Dr Windle Taylor at the next meeting of the Division.[1]

He also wished to confirm the Author's figure with regard to rural areas. Mr Forbes's own undertaking had laid down a sewerage works and had metered the water consumption. It had been found that the consumption per head per day was 14 gallons, so that it might be said that the sedimentary tanks were always in a state of septicity.

His own experience of storm-water tanks was that, except during the winter period, they rarely overflowed and gave an effluent into the river. He thought that when they did fill up the storm had passed and the time had come for the storm-water-tank liquor to be pumped back into the works for treatment. There was also the point that rain-water possessed what was termed "nascent oxygen," which was much more chemically active than the ordinary oxygen of water, and that attacked the impurities and rendered them innocuous very quickly.

The Chairman said that there was one point on which he thought there might be a certain amount of elaboration, namely, cost, since it affected controlling factors. He knew that the Government was very worried about the ever-increasing cost of sewerage and sewage-disposal works. The costs had rocketed during the past few years, and it was up to designers, consultants, and all concerned with the provision of new works to try to get the most efficient works at the minimum cost. If they failed to do that, the position was going to be really serious, particularly in rural areas where it might adversely affect the provision of new housing and the general improvement of living conditions.

*** **Mr R. F. Wills**, of Wellington, New Zealand, said that he was at present engaged on the design of a small sewage-purification works in

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[1] See Proceedings, Part III, vol. 2, December 1953.

*** This contribution was submitted in writing upon the closure of the oral discussion.—SEC. I.C.E.

that country, and outlined the various factors controlling the choice of process.

In comparing British and American design practice, it was noticeable that the British practice regarding sedimentation-tank design was more conservative than the American. Since conditions in New Zealand more closely resembled those in America, with high water-consumption and weak sewages, the American practice had been generally adopted there. Application of British practice would make works much more expensive, as pointed out by the Author when he had said that the cost of sedimentation of sewage from communities using large quantities of water was high in comparison with the percentage purification achieved. But the quality of the effluent was the main criterion in sedimentation-tank design, and the amount of suspended solids in the effluent from a tank treating a large volume of weak sewage with, say, 3 hours' detention at average flow, might be less than the amount in the effluent from a tank of similar size treating half the volume of a much stronger sewage. Consequently, the cost of sedimentation might be no greater.

Estimates taken out recently showed that, in New Zealand, activated sludge plants showed a saving in capital costs over the normal low-rate percolating filter plant for treating domestic sewage from populations above 10,000. But with the recent advances made in alternating double filtration, high-rate filtration, and re-circulation, it seemed that the percolating filter could compete favourable with activated sludge in area of land required and also in cost, for the treatment of domestic sewage.

The Author, in reply, thanked all those who had contributed to the discussion, and hoped that since the subject was so scantily treated in textbooks, on sewage treatment, the Paper would be of some value to both students and practitioners.

He agreed with Mr Townend that, with regard to wet-weather flows, the strength of the liquid discharged to the river was the true criterion, and that the terms 3 d.w.f. or 6 d.w.f. did not take sufficient account of possible variations in strength. Mr Townend's suggestion that the division of full from partial treatment should be based on pollution load per head of population was therefore logical. On the question of mechanization, the Author thought his introductory remarks would help to refute any suggestion by Mr Townend of timidity in that direction. He agreed with all those speakers who had pointed out the advantages of mechanization, but he had tried to emphasize in the Paper the need for discrimination in its application. Apart from that he was wholly in favour of mechanization, since in his opinion it was the only solution to the difficult problem of obtaining labour for the more unpleasant type of work connected with sewage treatment. In addition, its application could reduce capital expenditure in some instances, notably in sedimentation processes, where cleaning by hand always involved putting a tank out of operation, and

consequently an additional unit was required, whereas with mechanical cleaning the work could be done while the tank was in operation, and the additional unit was therefore unnecessary.

Both Mr Townend and Mr Lockett had disagreed with him on the suitability of the activated-sludge process as compared with percolating filters for the treatment of strong industrial sewages. Whilst he hesitated to disagree with such eminent authorities he still felt that most activated-sludge plants were less capable of producing consistently satisfactory results than were filter installations on such sewages. He did not think it was true to say that the activated-sludge process would solve all the problems involved in the treatment of strong sewages containing a high proportion of industrial wastes, but he thought that the manner in which it was applied at the West Middlesex works and at the works of his own Authority had gone a long way to prove that the process was far more robust and capable of satisfactorily treating such sewages than had at first been considered.

Mr Braine had raised several important points, one of the most interesting being the formula determining the economic size of sewers. Good gradients were very desirable for economy in maintenance and operation, as well as from the point of view of capital cost.

The Author agreed with Mr Braine's comments on the increasing difficulty of obtaining labour to do the dirty and disagreeable work connected with most of the older works in Great Britain, and he agreed that such labour should be replaced by machines wherever possible, provided that the replacement did not increase costs unduly. Mr Braine's revolutionary proposal for using humus tanks as storm-water tanks was very ingenious, but although some saving in capital cost might be achieved, he doubted whether it would prove so satisfactory in practice as separate tanks for the settlement of storm-water and filter effluent, particularly if both are mechanically cleaned. He agreed that at many of the older works there were tendencies to operate both kinds of tanks improperly, but in his view that was an operational rather than a design problem. By combining the two functions in one tank, the great advantage of storage was immediately lost. That advantage, which could only be obtained when *empty* tanks were available, had become of greater importance in recent years owing to the tendency towards fully separate systems of drainage, the effect of which had been to decrease the proportion of storm-water passing to the river without full treatment. The great advantage of storage was that full treatment could be given, without appreciable extra cost, to all storm-water so stored, with a corresponding reduction in river pollution. An added advantage lay in the fact that the flow so stored, namely, the first flushes at the onset of storm-water flows, were nearly always very strong and might be extremely foul. Mr Braine's scheme appeared to provide no adequate compensation for the sacrifice of those advantages, and in the Author's opinion the dilution of storm-

water with purified effluent was no substitute for the complete treatment to which storage could and should lead.

He agreed with Mr Braine that considerable saving in capital cost could be achieved by constructing secondary settlement tanks with flat bottoms, and in his opinion the design of sludge-scraping mechanism described by Mr Braine had great possibilities. That type of scraper had been introduced about 50 years ago, but had not been particularly successful because the advantages of continuous sludge removal, and the possibilities of that design of blade for achieving it had not been generally appreciated at that time.

He agreed with Mr Calvert that if the figures given in the Paper for the proportional areas required for purification processes were interpreted as being representative of the areas of treatment works as a whole, they were misleading, and he wished to correct such an impression if it had been given in the Paper. Mr Calvert had stated that in his opinion there was no need for complete removal of solid matter in cases where final effluent was used for irrigation, but whilst the Author agreed that the humus had a value as a fertilizer and soil conditioner, he had to draw attention to the danger to health arising from the consumption of raw crops. In some parts of the world, notably in South Africa, the transmission of parasitic diseases in that manner had been found to be serious. Where the crops grown could be used for fattening livestock, those dangers were eliminated, and irrigation by means of partially purified sewage became advantageous.

He was afraid he could not yet express an opinion on Mr Calvert's question about the minimum economic size for schemes incorporating vacuum filtration of sludge. So many factors were involved that each case would have to be considered on its merits, but in his opinion the process could only be economically applied to large and moderately large schemes.

He had not mentioned in the Paper a subject touched upon by Mr Lockett, namely, the modern use of synthetic detergents and their effects on sewage-treatment processes generally and on sewage effluents, because it was a problem for chemists rather than engineers. It was also a serious national problem and it had yet to be solved. He was not in a position to say how it was going to affect sewage-treatment processes and the choice of processes. There was, however, no doubt that it was going to add to the difficulties of sewage treatment. He felt himself that the solution should be sought at the manufacturers' end and not at the sewage works.

He agreed with Mr Manning that dispersal in water was fundamentally a sewage-treatment process, and that it was not therefore strictly correct to refer to it, as he had done in the Paper, as disposal of sewage without treatment. It was really leaving the treatment to Nature, as distinct from the provision of artificial works in which the processes could be regulated and controlled.

With regard to standards for sewage effluents, the only tendencies of which the Author was aware were towards higher standards, and in his opinion such tendencies were bound to persist, though he agreed with Mr Manning and other speakers that each case should be considered on its merits, the river being used for purification to the fullest extent compatible with local conditions and requirements.

Mr Manning had given an interesting example of site conditions being a major factor in the choice of sewage-treatment processes. One could, no doubt, find other examples of a similar kind, but the Author still considered that in the majority of cases, site conditions could not be regarded as of the same importance in the determination of the processes to be used as the major factors he had cited, which were always of prime importance.

On the question of labour, Mr Manning had raised a point which he had tried to stress in the Paper. He agreed with Mr Manning that, where skilled labour was available, one need not cling to simplicity, but in less-developed parts of the world one would probably have to do work both of construction and of operation and maintenance with what would be called very unskilled labour in Great Britain. In those circumstances, simplicity in design was essential.

Mr Forbes had raised several very interesting points on which he did not think he need comment very extensively. The efficiency of sedimentation was also dependent upon surface area, the shape of the tank, and in particular upon the design of inlets and outlets. He agreed that mechanization for the sake of releasing land would in certain instances be of some importance, but he still thought that the choice of mechanization should depend on more important factors than that, particularly capital and maintenance costs.

On the question of the discharge of sewage into the sea and of salt water being bacteriocidal, he could give no authoritative opinion, but it was a matter about which there had been very serious thought in the United States, and it had been found that pathogenic organisms did survive in the sea for a certain period and that there would be some danger to health in bathing off coasts where the sea was contaminated with sewage or sewage effluents ; nor could he comment on the point about nascent oxygen and its value in solution in rain-water, which again was a matter for the chemists, but he agreed with Mr Forbes' comments on storm-water tanks and their operation.

The closing date for correspondence on the foregoing Paper has now passed, and no contributions other than those already received at the Institution can be accepted.—Sec. I.C.E.

Paper No. 5894

"Further Research in Reinforced Concrete, and its Application to Ultimate Load Design"*

by

Professor Arthur Lemprière Lancey Baker, B.Sc.Tech., M.I.C.E.

(*Ordered by the Council to be published with written discussion.*)†

SYNOPSIS

The Paper describes, with examples, a trial-and-adjustment plastic-hinge method of designing frames many times statically indeterminate. The rotations of plastic hinges under load are conveniently calculated and adjusted by tabulation and the use of influence coefficients. When the number of hinges is high, the work of calculating influence-coefficient values may be performed by drawing bending-moment diagrams on "cards" which may be "packed and played" according to rules, the process of integration thereby being systematic. Alternatively, general expressions may be used which have been determined for the case of a building frame having n bays and m storeys. Rules for determining first trial values of plastic moments and making adjustments to obtain an economic ultimate-load solution or an approximate elastic solution are given.

Results of various tests are reported from which are derived safe limiting values of the factors affecting the strength and available deformation of plastic hinges. The trial-and-adjustment method described might be used to derive simple formulae for the ultimate bending strength of the members of the more common forms of building frame. No reference has been made to shear strengths, but it is anticipated that the formation of plastic hinges will not cause serious weakening, particularly when full shear reinforcement is provided.

Results are given of the tests in regard to the ultimate strength in bending of reinforced-concrete beams, prestressed-concrete beams, and cylindrical shells, which were described in Structural Paper No. 26.

Introduction

The theory of plastic hinges provides a practical design method for frameworks which are many times statically indeterminate, and a means of establishing simple design formulae for a wide range of structures subject to both vertical and horizontal loads. It aims at selecting hinge positions and plastic-moment values, and the relative stiffnesses of members, so that the distribution of bending moments prior to failure is economic and so that permissible strains and deflexions under working load are not exceeded. The procedure is first to assume the location of plastic hinges and values of plastic moments which appear to provide the best distribution of ultimate

* This Paper is the Addendum to Structural Paper No. 26, J. Instn Civ. Engrs, vol. 35, p. 262 (Feb. 1951).

† Correspondence on this Paper should be received at the Institution by the 15th December, 1953, and will be published in Part III of the Proceedings. Contributions should be limited to about 1,200 words.—Sec. I.C.E.

bending moments. A trial-and-adjustment process is then followed until conditions which indicate a correct choice of hinge positions are satisfied and so that under working load excessive strain is avoided. The method provides a means of establishing simple design formulæ for ultimate-wind and vertical-load bending moments for building frames which are in common use. Concrete sections are not generally varied from the original assumptions—the variation of steel areas which can easily be made provides the chief means of making local adjustments to the bending strength.

Notation (*suitable for building-frames or any framework composed of a large number of rectangular bays*)

Framework bays are numbered from left to right 1, 2, . . . p . . . n, $n + 1$, the last being an imaginary bay. (See *Figs* 2 and 9.)

Framework storeys are numbered from bottom to top 1, 2, . . . $r - 1$, r, $r + 1$, . . . m. In each bay there are three hinges a, b, c, generally located for wind on the left, as shown in *Fig*. 9 (r).

Each hinge is denoted by indicating in order its bay number, storey number, and type. Thus $p - r - b$ denotes hinge type b in bay p in the rth storey.

$\bar{X}_{p-r-b}$ denotes the plastic-moment value of hinge $p - r - b$.

X_{p-r-b} ,, the unknown moment of resistance at hinge $p - r - b$ for the elastic condition.

M_{pr} ,, the maximum value of the external bending moment on the upper beam of the rth storey in the pth bay (a parabolic distribution has been assumed in the general expressions).

m_r ,, the maximum value of the external sway bending moment in the rth storey, for the hinge system indicated in *Figs*. 2 and 9.

θ_{p-r-b} ,, the rotation at hinge $p - r - b$ when all plastic-hinge moments and external bending moments act. The term "rotation" of a plastic hinge refers to the relative rotation of ends of members adjacent to the hinge; that is, the difference in slope of the ends of the members or the angle of discontinuity.

θ_{ik} refers to rotations such as θ_{p-r-b} in a general sense.

E denotes modulus of elasticity of concrete.

I_{pr} ,, moment of inertia of the upper beam in the pth bay of the rth storey.

J_{pr} ,, the moment of inertia of the left-hand column of the pth bay in the rth storey.

h_r ,, the height of the columns in the rth storey.

l_{pr} ,, length of the upper beam in the pth bay of the rth storey.

Basic Assumptions

(1) When a framework which is n times statically indeterminate is increasingly loaded throughout, n plastic hinges form before failure occurs, and the structure becomes statically determinate.

(2) The members of the framework between the plastic hinges remain elastic and do not yield when the ultimate load as defined in (3) is applied.

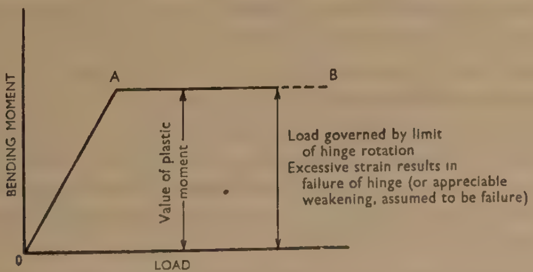
(3) The load applied when the nth plastic hinge forms is the ultimate load.

(4) The safe limiting values of the factors which govern the available rotations of plastic hinges and the deformations of the elastic members cover the errors involved in assumptions (2), (5), and (6), and also those arising from variations in EI values resulting from cracking and changes in the position of the neutral axis over the length of the members.

(5) The plastic hinges are concentrated at points.

(6) Throughout the framework under increasing load, the relation between load and moment of resistance follows a straight-line law such as is indicated by OA in *Fig. 1*, except at the hinge points where, after the plastic moment of resistance has developed, a horizontal line such as AB is followed.

Fig. 1



IDEALIZED BENDING-MOMENT/LOAD CHARACTERISTICS OF PLASTIC HINGE

(7) The value of the bending moment at A, *Fig. 1*, is governed by the ultimate strength of the concrete in over-reinforced beams or columns and by the yield strength of the steel in under-reinforced beams.

(8) The length of the line AB, *Fig. 1*, is limited by the ultimate strain of the concrete, and also the ultimate strength in under-reinforced beams.

(9) If the cross-section of the framework in any part for practical convenience is made stronger than the minimum section required by the calculations, the ultimate strength of the whole framework is not reduced, although the positions of the plastic hinges may be altered.

(10) An adequate factor of safety against failure by repeated overloading on parts of the structure is provided when the whole framework is designed for the worst cases of full load acting once, and calculations include a normal value of the load factor of safety.

Trial-and-Adjustment Design of Reinforced-Concrete Frameworks by the Theory of Plastic Hinges

Basic Assumptions—Explanatory Notes

(1) The structure becomes statically determinate since the values of the moments of resistance of the plastic hinges are known. In some cases, when full ultimate load is applied, local failure may occur before plasticity develops at all the assumed hinge points. Unknown elastic moments of resistance must then be assumed to act at such points and the elastic equations accordingly adjusted. The procedure is demonstrated later by an example.

(3) Framework members between plastic hinges will be designed to resist, without yield, the bending moment applied to them when the nth hinge has formed; the actual ultimate load will therefore exceed the assumed ultimate load by a small margin related to the excess of actual bending strength provided for practical convenience above the theoretical minimum requirement for avoiding yield between the plastic hinges.

(5) Plasticity develops under increasing load at points of maximum stress which are generally arranged by adjustment of reinforcement to occur at supports, and spreads longitudinally until sufficient yield over sufficient length provides the required rotation or angle of discontinuity between the adjacent members. Sufficient width of support with limited encroachment on the span must be provided, so that the available hinge rotation as calculated from safe limiting values of l_p, s_p, or s_d [1] is adequate. For the purpose of calculating bending moments, a hinge acting at a point is a convenient idealized conception of what is actually a short plastic length of a member at a support equivalent to the hinge, since it provides the same local change of slope or angle of discontinuity between the ends of the adjacent elastic members. It is assumed that the framework members span between the points, and this is safe, since thereby the calculated values exceed the actual values of hinge rotations. When hinge rotation is mainly governed by steel yield, local bond slip accompanied by cracking can extend the plastic zone a little beyond the support. The local distribution of stresses then virtually causes a spreading of the peak of the idealized bending-moment diagram. Hinge rotation in columns which are stressed entirely in compression is governed by compressive yield of the concrete within closely spaced stretched binders, and again the local distribution of stresses in the plastic range virtually causes a spreading of the peak of the idealized bending-moment diagram.

(6), (7), (8) *Fig. 1* indicates the assumed idealized elasto-plastic properties of the framework members and hinges, upon which calculations are based. For particular grades of concrete and steel, assumed safe limiting values of E, ultimate strength and strain of concrete, and yield

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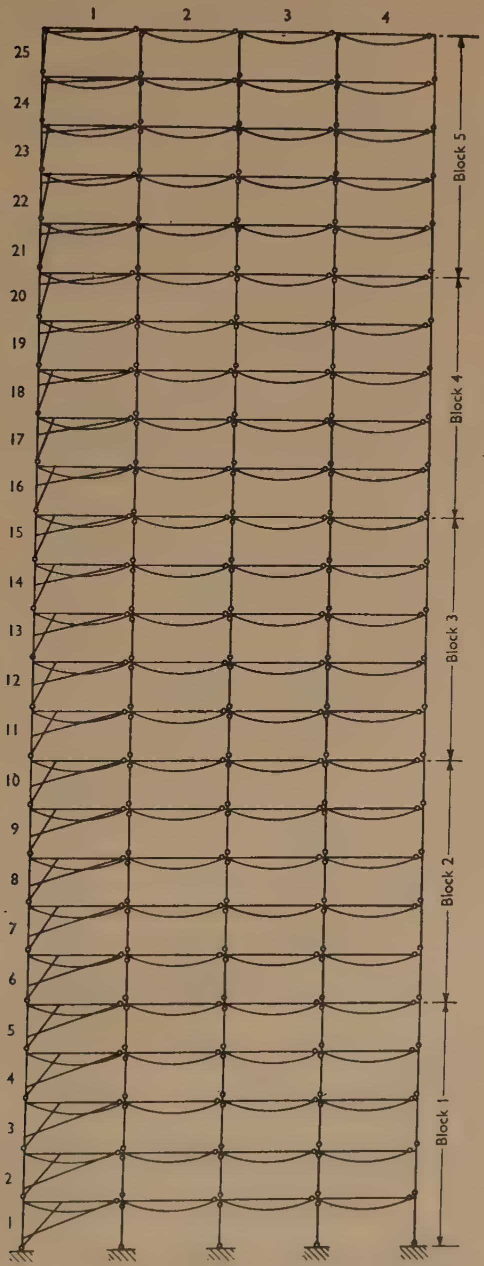
[1] The references are given on p. 308.

Fig. 2



ASSUMED HINGE POSITIONS IN 25-STOREY, 4-BAY BUILDING FRAME

strength of steel must be related to specimen test results when calculating elastic deformations of members, and ultimate strengths and rotations of hinges.

(9) In theory, it is possible that a local increase of strength causing a large local increase of stiffness might so alter the distribution of bending moments in the framework that it would be weakened. Such an abnormal condition is not likely to occur in practice.

(10) In some special cases, repetition of overload smaller in value than ultimate load applied only once and following a particular cycle of application, might cause accumulative plastic strain, and so failure, after several applications. It can generally be assumed that there is less chance of slight overload being repeated many times in the worst possible way than the full ultimate load occurring once. The latter case, therefore, is a sound criterion for ultimate strength. In cases where load such as wind load is liable to be applied in alternative directions, one direction may be considered at a time, and the sections of the framework calculated accordingly, but the actual members must be made strong enough for either case.

Selection of Hinge Positions and Bending-Moment Values

Some experience of the elastic theory of frameworks is required in order to select satisfactorily first trial positions and values of hinges. Hinges should be assumed to develop at sections at which maximum bending moments occur under elastic conditions, and to have plastic-moment values which produce an economic distribution of bending moments. In a building frame, the positions indicated in *Fig. 2* with respect to vertical and wind load from the left will be found to be suitable for many cases, since these occur at sections where wind moments and vertical load moments have maximum values and combine together. The first-trial values of the plastic moments of resistance of these hinges producing an economic distribution of bending moments may be obtained as the sum of the bending moments—cases (*a*), (*b*), and (*c*)—as shown in *Figs 3*.

Fig. 3 (*a*) Vertical load, inside beams. Support bending moments equal mid-span bending moments.

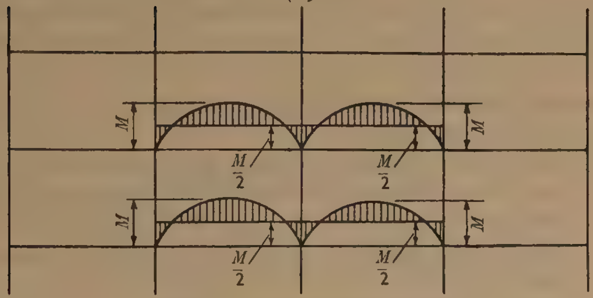
Fig. 3 (*b*) Vertical load, end-span beams. Assume that the supports are first fixed as in the well-known Hardy-Cross method. The support bending moment will then be $\frac{2}{3}M$ for uniformly distributed load. Distribute the fixed-support bending moment between the beam and columns in proportion to their stiffness.

M_u denotes restraint bending moment of upper column
M_l ,, restraint bending moment of lower column
h_u and h_l denote the heights of the upper and lower columns
J_u and J_l ,, the moments of inertia of the upper and lower columns
l_B denotes the length of the end-span
I ,, the moment of inertia of the end beam

Then $M_u = \frac{2}{3}M \dfrac{\dfrac{J_u}{h_u}}{\dfrac{J_u}{h_u} + \dfrac{J_l}{h_l} + \dfrac{I}{l_B}}$

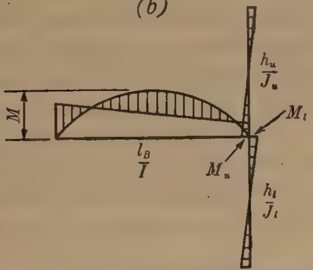
Figs 3

(a)



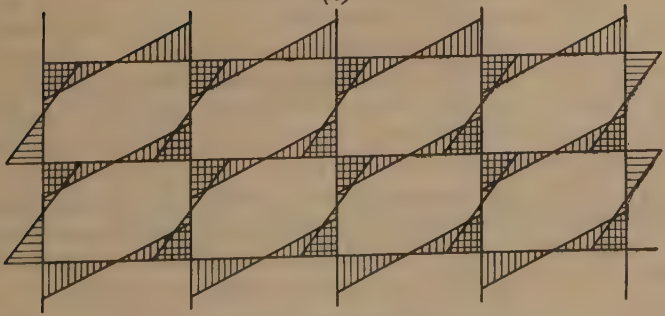
INSIDE BEAMS: FIRST TRIAL BENDING MOMENTS
Mid-span moments = support moments

(b)



END-SPAN BEAMS: FIRST TRIAL BENDING MOMENTS

(c)



FIRST TRIAL WIND BENDING MOMENTS

TYPICAL FIRST TRIAL BENDING-MOMENT DISTRIBUTION

$$M_l = \tfrac{2}{3}M \frac{\frac{J_l}{h_l}}{\frac{J_u}{h_u} + \frac{J_l}{h_l} + \frac{I}{l_B}}$$

The beam support bending moment $= M_u + M_l$.
In a top storey, $h_u = 0$, and is omitted.

Fig. 3 (*c*) Wind Load—Assume that the points of contraflexure in the columns and the beams are central and that the total wind shear is divided among the columns in proportion to the column stiffnesses. Bending-moment values can then be found by simple statics.

The assumed positions, plastic-bending moments, and deformability values of hinges are satisfactory if :—

(1) The sum of the rotations at each hinge due to loads and all plastic-hinge moments obtained from the general expressions given below, is negative in value (θ then is positive in value owing to the negative sign of θ in the general elastic equations).[1]

(2) The value of the resultant bending moments for ultimate load, as defined under the heading "Basic Assumptions" at all sections between the plastic hinges, is within the elastic range.

(3) The value of the rotation at each hinge does not exceed an appropriate safe limiting value for that hinge, in order to avoid premature crushing of the concrete.

(4) At working load, elastic conditions obtain at all hinges and the strains are small enough to avoid wide cracks and large deflexions. (The distribution of bending moments for this purpose need only be determined approximately.)

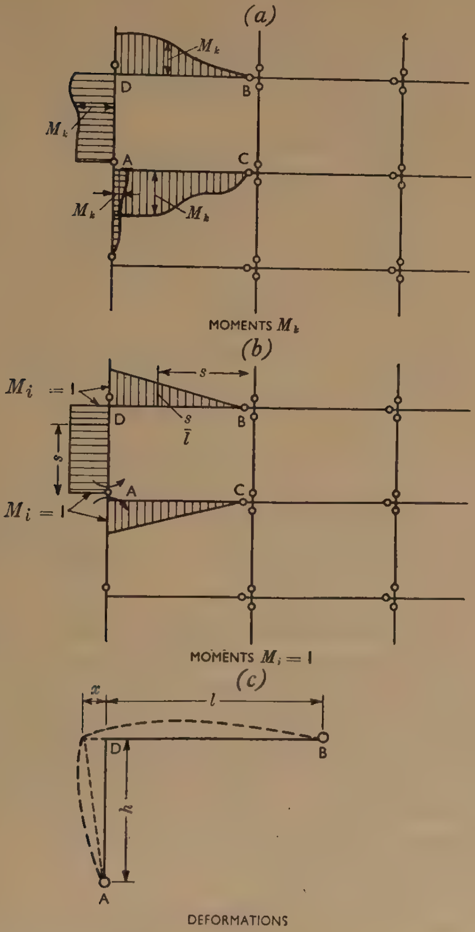
General Expression for Rotation of Plastic Hinges

The simplest and most comprehensive method of deriving a general expression for the rotation of a plastic hinge is by use of the principle of virtual work (p. 266 of reference 1, and reference 2). For those who prefer to approach deformation problems by slope-deflexion methods, the following derivation giving the same result may be more satisfactory. A thorough study of the derivation and use of general elastic equations is, however, recommended as given in the above references and reference 4.

Fig. 4 (*a*) shows moments M_k which might be due to external load or internal plastic-hinge restraints acting on the members adjacent to any hinge A (whose plastic moment of resistance is M_i) of a frame made statically determinate by assuming a sufficient number of plastic hinges.

Fig. 4 (*b*) shows moments due to $M_i = 1$ acting on either side of hinge A.

Figs 4



MOMENT AND DEFORMATION DIAGRAMS FOR SLOPE-DEFLEXION TREATMENT OF HINGE ROTATIONS

Fig. 4 (*c*) shows the deformations due to moments M_k and the sway x.

The point D sways an amount x to the left until the slope of the column at D equals the slope of the beam, and therefore so that :

$$\frac{x}{h} = \frac{1}{EI}\int M_k \frac{s}{h} ds + \frac{1}{EI}\int M_k \frac{s}{l} ds,$$

by Mohr's rule, and well known expressions for the slope at the support of a beam.

The slope at A, of the column AD, caused by deformation of the column and the sway x, is given by :

$$\text{slope} = \frac{1}{EI}\int M_k \frac{(h-s)}{h} ds + \frac{x}{h}$$

Substituting for $\frac{x}{h}$ obtained above :

$$\text{slope} = \frac{1}{EI}\int M_k \cdot 1 \cdot ds + \frac{1}{EI}\int M_k \frac{s}{l} ds$$

$$\text{but } M_i = 1 \text{ along AD and } M_i = \frac{s}{l} \text{ along DB}$$

$\therefore$ slope at A $= \frac{1}{EI}\int M_i M_k \,.\, ds.$ When M_k acts on members AD and DB the product $M_i M_k$ being integrated along those members.

Similarly the slope of AC at A is $\frac{1}{EI}\int M_i M_k \,.\, ds.$ for the member AC.

Thus it can be stated generally that the rotation of the hinge at A (that is,$-\theta_{ik}$) caused by M_k is $\frac{1}{EI}\int M_i M_k \,.\, ds$ for the members on which $M_i = 1$ acts.

Rotations of hinges can therefore be determined by plotting on the members of the frame separate diagrams of the moments due to load and due to unit equal and opposite moments acting at each hinge. The resultant rotation at any hinge when all loads act, is the sum of the rotations due to the external load moments and the known internal plastic or unknown elastic moments acting at each hinge including the equal and opposite moments acting at each side of the hinge itself. *Fig. 5* shows a Table giving values of $\int M_i M_k \,.\, d_s$ for common cases.

Convention of Signs

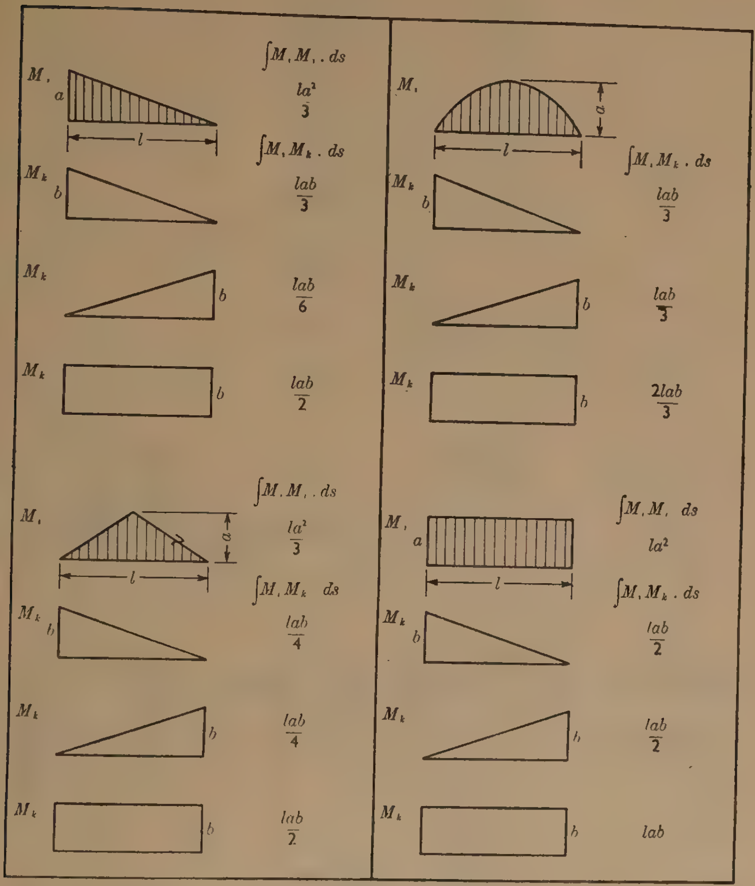
Bending-moment diagrams are plotted on the tension side of members. When M_i and M_k are on the same side of a member the integration value is positive, and negative when M_i and M_k are on opposite sides, since either M_i or M_k must then be negative in sign.

As already stated, rotations at hinges are negative when opposed to the internal couple acting at the hinge, so that θ must be positive in value when found from the general expression in which $-\theta$ is assumed to be the rotation value, if there is an angle of discontinuity.

General Method of Checking and Adjusting Plastic Hinges

The framework in *Figs 6* may be used as an example to demonstrate a general procedure for checking rotations and adjusting values of assumed bending moments at plastic hinges. The example is intended mainly to

Fig. 5



TYPICAL $\int M_1 M_k \, ds$ VALUES

demonstrate the method, and does not show as well as the second example how effective the method can be in dealing with cases many hundreds of times statically indeterminate. The beams support uniformly distributed load, and horizontal forces act from the left, causing the sway moments shown. Hinge points are assumed as shown in *Fig. 6* (*a*), that is, at points of guessed maximum bending moment for elastic conditions. The loads then cause bending moments M and m as in *Figs 6* (*b*) and (*c*). Assumed values of the plastic-hinge moments are approximately equal to values obtained as already explained, and are the ringed figures in *Figs 6*.

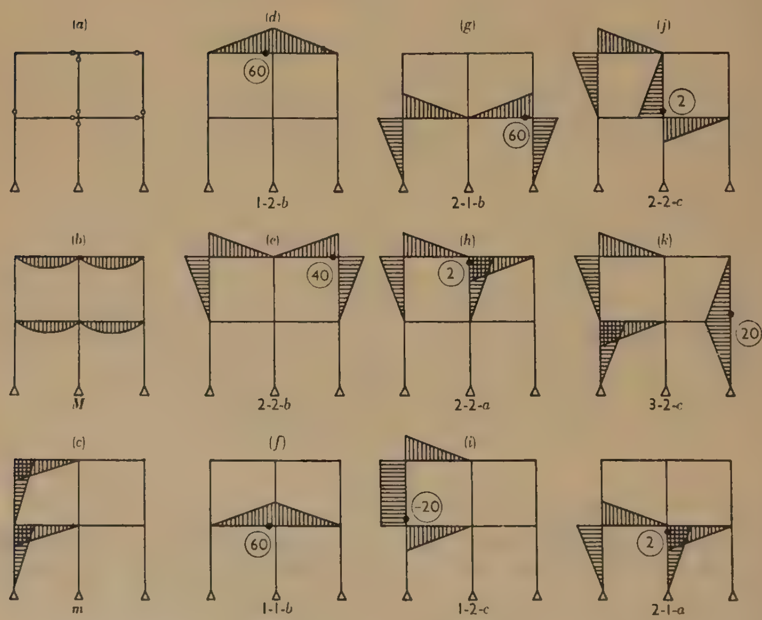
$$M = 120 \text{ units}$$

$$m = 12 \text{ units}$$

$$\bar{X}_{1-2-b} = 60 \text{ units}$$
$$\bar{X}_{2-2-b} = 40 \text{ units}$$
$$\bar{X}_{1-1-b} = 60 \text{ units}$$
$$\bar{X}_{2-1-b} = 60 \text{ units}$$
$$\bar{X}_{2-2-2} = 2 \text{ units}$$
$$\bar{X}_{1-2-c} = -20 \text{ units}$$
$$\bar{X}_{2-2-c} = 2 \text{ units}$$
$$\bar{X}_{3-2-c} = 20 \text{ units}$$
$$\bar{X}_{2-1-a} = 2 \text{ units}$$

$$\frac{l}{EI} = \frac{h}{EJ} \text{ for all members} = 6 \text{ units}$$

Figs 6



Bending-Moment Cases of Frame Example, Nine Times Statically Indeterminate

The resultant rotation at any plastic hinge at which $M_i = 1$ acts due to all moments acting as shown in *Figs 6* (*b*) to (*l*) can be found and adjusted by successively applying the general expression $\int \frac{M_i M_k \cdot ds}{EI} = -\theta_{ik}$, first for unit bending moment acting at the hinge itself and at any other hinge

TABLE 1.—TRIAL AND ADJUSTMENT TABLE FOR FRAME EXAMPLE NINE TIMES STATICALLY INDETERMINATE

(2)			HINGES																																			
			1-2-*b*				2-2-*b*				1-1-*b*				2-1-*b*				2-2-*a*				1-2-*c*				2-2-*c*				3-2-*c*				2-1-*a*			
P	A_P	A_E	U	P	A_P	A_E	U	P	A_P	A_E	U	P	A_P	A_E	U	P	A_P	A_E	U	P	A_P	A_E	U	P	A_P	A_E	U	P	A_P	A_E	U	P	A_P	A_E	U	P	A_P	A_E
120			−4	$\boxed{-480}$			−4	$\boxed{-480}$			−4	$\boxed{-480}$			−4	$\boxed{-480}$			—	—			—	—			—				—	—			—	—		
12			−1	$\boxed{-12}$			−4	$\boxed{-48}$			−1	$\boxed{-12}$			−4	$\boxed{-48}$			−4	$\boxed{-48}$			−3	$\boxed{-36}$			−4	$\boxed{-48}$			—	—			−4	$\boxed{-48}$		
60		30	4	240		120	2	120		60	—	—		—	—	—		—	−1	$\boxed{-60}$		$\boxed{-30}$	1	60		30	1	60		30	1	60		30	—	—		—
40		4	2	80		8	8	320		32	—	—		—	—	—		—	3	120		12	5	200		20	4	160		16	3	120		12	—	—		—
60		30	—	—		—	—	—		—	4	240		120	2	120		60	—	—		—	−1	$\boxed{-60}$		$\boxed{-30}$	−1	$\boxed{-60}$		$\boxed{-30}$	−1	$\boxed{-60}$		$\boxed{-30}$	−1	$\boxed{-60}$		$\boxed{-30}$
60			—	—			—	—			2	120			8	480			—	—			−2	$\boxed{-120}$			−1	$\boxed{-60}$			−6	$\boxed{-360}$			3	180		
2		−8	1	2		$\boxed{-8}$	3	6		$\boxed{-24}$	—	—		—	—	—		—	8	16		$\boxed{-64}$	5	10		$\boxed{-40}$	3	6		$\boxed{-24}$	4	8		$\boxed{-32}$	—	—		—
−20		8	1	$\boxed{-20}$		8	5	$\boxed{-100}$		40	−1	20		$\boxed{-8}$	−2	40		$\boxed{-16}$	5	$\boxed{-100}$		40	10	$\boxed{-200}$		80	5	$\boxed{-100}$		40	7	$\boxed{-140}$		56	−2	40		$\boxed{-16}$
2	−6	−10	1	2	−6	$\boxed{-10}$	4	8	−24	$\boxed{-40}$	−2	$\boxed{-4}$	12	20	−1	$\boxed{-2}$	6	10	3	6	−18	$\boxed{-30}$	5	10	−30	$\boxed{-50}$	8	16	−48	$\boxed{-80}$	4	8	−24	$\boxed{-40}$	2	4	$\boxed{-12}$	$\boxed{-20}$
20		12	1	20		12	3	60		36	−1	$\boxed{-20}$		$\boxed{-12}$	−6	$\boxed{-120}$		$\boxed{-72}$	4	80		48	7	140		84	4	80		48	12	240		144	−4	$\boxed{-80}$		$\boxed{-48}$
2	−6	10	—	—	—	—	—	—	—	—	−1	$\boxed{-2}$	6	$\boxed{-10}$	3	6	−18	30	—	—	—	—	−2	$\boxed{-4}$	12	$\boxed{-20}$	1	2	−6	10	−4	$\boxed{-8}$	24	$\boxed{-40}$	8	16	$\boxed{-48}$	80
				−512	−6	148		−628	−24	168		−518	18	140		−650	−12	100		−208	−18	100		−420	−18	214		−268	−54	144		−568	0	242		−188	−60	80
				344		−18		514		−64		380		−30		646		−88		222		−124		+420		−140		324		−134		436		−142		240		−114
$-\theta_{ik}$				−168		130		−114		104		−138		110		−4		12		+14		−24		0		+74		+56		+10		−132		100		+52		−34
$-\theta_{ik}$				−174				−138				−120				−16				−4				−18				+2				−132				−8		

TABLE 3.—TRIAL AND ADJUSTMENT TABLE FOR BENDING-MOMENT VALUES OF CRITICAL HINGES IN BUILDING FRAME 289 TIMES STATICALLY INDETERMININATE

				(1) 1–r–b			(p) 2–r–b			(p) 3–r–b		
	P	A_P	A_E	U	A_P	A_E	U	A_P	A_E	U	A_P	A_E
(1) 1–r–b	5		2	8		16	2		4			
(p) 2–r–b	5		2	2		4	8		16	2		4
(p) 3–r–b	5		2	—		—	2		4	8		16
(n) 4–r–b	4		2	2		4	—		—	2		4
(p) 2–r–a	·1			−2			−2					
(p) 3–r–a	·1			2			−4			−2		
(n) 4–r–a	·1			2						−4		
(1) 1–r–c	2·5	−2	−2·5	2	−4	$\boxed{-5}$						
(p) 2–r–c	·1			2								
(p) 3–r–c	·1			2								
(p) 4–r–c	·1			2								
($n+1$) 5–r–c	2·5		2	2		4						
Total A_P					−4							
Total A_E						23			24			24
1st trial	$-\theta_{ik}$			−22			−20			−20		
Final trial	$-\theta_{ik}$			−26			−20			−20		

	End hinge (n) 4–r–b			(p) 2–r–a			(p) 3–r–a			End hinge (n) 4–r–a		
	U	A_P	A_E	U	A_P	A_E	U	A_P	A_E	U	A_P	A_E
(1) 1–r–b	2		4	−2		$\boxed{-4}$	2		4	2		4
(p) 2–r–b	—		—	−2		$\boxed{-4}$	−4		$\boxed{-8}$	—		
(p) 3–r–b	2		4				−2		$\boxed{-4}$	−4		$\boxed{-8}$
(n) 4–r–b	12		24	6		12	6		12	4		8
(p) 2–r–a	6			12			6			6		
(p) 3–r–a	6			6			12			6		
(n) 4–r–a	4			6			6			12		
(1) 1–r–c	7	−14	$\boxed{-17}$	7	−14	$\boxed{-17}$	7	−14	$\boxed{-17}$	7	−14	$\boxed{-17}$
(p) 2–r–c	6			5			6			6		
(p) 3–r–c	6			6			5			6		
(p) 4–r–c	6			6			6			4		
($n+1$) 5–r–c	5		10	6		12	6		12	6		12
Total A_P		−14			−14			−14			−14	
Total A_E			25			−1			−1			−1
1st trial	−12			11			11			12		
Final trial	−26			−3			−3			−2		

	(1) 1–r–c			(p) 2–r–c			(p) 3–r–c			(p) 4–r–c			End hing ($n+1$) 5–r–c	
	U	A_P	A_E	U	A_P	A_E	U	A_P	A_E	U	A_P	A_E	U	A_P
(1) 1–r–b	2		4	2		4	2		4	2		4	2	
(p) 2–r–b														
(p) 3–r–b														
(n) 4–r–b	7		14	6		12	6		12	6		12	5	
(p) 2–r–a	7			5			6			6			6	
(p) 3–r–a	7			6			5			6			6	
(n) 4–r–a	7			6			6			5			6	
(1) 1–r–c	14	−28	$\boxed{-35}$	7	−14	$\boxed{-17}$	7	−14	$\boxed{-17}$	7	−14	$\boxed{-17}$	11	−22
(p) 2–r–c	7			12			6			6			6	
(p) 3–r–c	7			6			12			6			6	
(p) 4–r–c	7			6			6			12			6	
($n+1$) 5–r–c	11		22	6		12	6		12	6		12	14	
Total A_P		−28			−14			−14			−14			−22
Total A_E			5			11			11			11		
1st trial	24			11			11			11			6	
Final trial	−4			−3			−3			−3			−16	

where M_k acts, and for maximum load bending moment assumed to have unit value, then by multiplying each rotation for unit bending moment by each bending-moment value.

When bending moment X_k acts, $-\theta_{ik} = X_k \int \frac{M_i M_k \, . \, ds}{EI}$, or when maximum load moment M acts, $-\theta_{ik} = M \int \frac{M_i M_1 \, . ds}{EI}$, where M_1 indicates bending moments of maximum value unity having the same distribution as M.

The trial and adjustment is conveniently tabulated as shown in Table 1. The various columns of the Table are as follows :—

(1) : The load moments and hinges whose plastic-moment values are to be assumed and adjusted.

(2) : P Values of load moments and first trial plastic-hinge moments.

A_P Adjustment to P for full plastic-hinge condition.

A_E Adjustment to P plus final A_P for elastic condition.

$1-2-b$: U Values of rotations at hinge $1-2-b$ for unit moment acting.

P Values of rotations at hinge $1-2-b$ for load moments or full plastic-hinge moments acting, that is to say, (2) : P times U.

A_P Adjustment of rotation at hinge $1-2-b$ for adjustment, (2) : A_P, that is to say, $2 : A_P$ times U.

A_E Adjustment of rotation at hinge $1-2-b$ for adjustment, (2) : A_E, that is, $2 : A_E$ times U.

Remaining columns are similar to $1-2-b$ for the remaining hinges.

The totals indicate first and final total trial rotation values for each hinge and the approximate rotation adjustment necessary for the elastic condition, that is, for zero resultant rotation.

Procedure for Computing Rotations for Unit Moments Acting

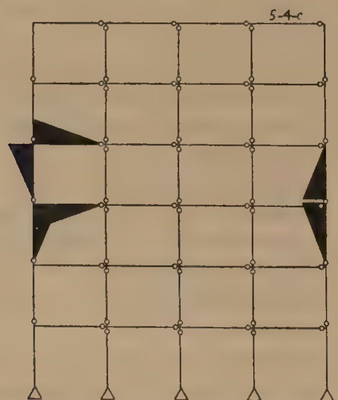
Referring to *Figs 6*, which show the various cases of bending moments for a framework which has been made statically determinate by the development of plastic hinges. The members are deformed elastically between the plastic hinges by the action of the load moments M and m, and equal and opposite moments at each of the nine plastic hinges, each considered at first to have maximum value unity. Each hinge is considered in turn and the rotations entered under the appropriate U columns of Table 1. The rotations are computed by applying the general expression $-\theta_{ik} = \int \frac{M_i M_k \, . \, ds}{EI}$ to each hinge, M_i being the bending moments due to unit moment acting at the hinge, and M_k being :

(1) the load moments (assumed to have unit maximum value),
or (2) unit moment acting at the other hinges,
or (3) unit moment acting at the hinge itself.

The integrations are readily determined by use of the computation Table *Fig. 5.*

Until experience has been gained, and to facilitate the work in cases which are many times statically indeterminate, it is a great help to plot diagrammatically each separate case of bending moment on a separate piece of paper about the size of a playing card with an index mark, so that there is a card for each hinge moment and each kind of load moment, for example, M and m. *Fig. 7* indicates a typical " card ". Although only a knowledge

Fig. 7



TYPICAL " STRUCTURE PATIENCE " CARD

of simple statics is required, it may at first be found difficult to plot the bending-moment diagrams correctly. The reactions at supports corresponding to each hinge bending moment must be considered. They may be vertical supports in the case of beam hinges, or horizontal supports provided by sway resistance moments in the left-hand bay in the case of column hinges. These sway moments will have unit maximum value. (See *Figs 9*). Integrations are then determined as follows (the process might well be termed " structure patience ") :

(1) If there are m storeys and n bays, arrange the " cards ", except those for moments M and m, in 3_n packs, each pack containing only (a), (b), or (c) cards of the same bay. The cards in each pack should be arranged in order starting from the bottom storey. Thus for bay p there are three packs containing cards in the following order :—

Pack (a) $p - 1 - a, p - 2 - a,$ . . . . $p - m - a$
Pack (b) $p - 1 - b, p - 2 - b,$ . . . . . $p - m - b$
Pack (c) $p - 1 - c, p - 2 - c,$ . . . . . $p - m - c$

(2) To compute the resultant rotation at each hinge, play each card except the M and m cards (that is, load bending moment) in turn and play against it each other card in turn, the card itself, and the M and m cards. Commence with the first bay working through pack (a), (b), (c), then the second bay, and so on.

(3) Whenever a pair of cards have bending-moment diagrams on the same framework members, a rotation occurs for the hinge under consideration, and the θ_{ik} value is computed using the table in *Fig. 5*, multiplying by the appropriate $\frac{l}{EI}$ value, and the result entered against the appropriate hinge and bending-moment in the U-column of a table such as Table 1.

(4) The rotation due to the moment acting at the hinge itself must also be included, so that each card must be played against itself, as already mentioned in (2) above.

It will be found that inside beam hinges are rotated only by adjacent hinges and load moments, whilst end beam hinges and column hinges are rotated by all other column hinges in the same storey and some in adjacent storeys. The arrangement of the cards which has been suggested readily indicates, for any particular hinge, the other hinges which influence its rotation, that is, the cases of M_k not giving zero values of $\int M_i M_k \, . \, ds$.

Adjustment of Hinge Moment Values

Referring again to *Figs 6* and Table 1, when the " U " columns have been completed the rotations for the full-load moments and hinge moments are computed and entered in the P-columns which are then added up giving the first trial values of resultant θ_{ik} for each hinge. Adjustments to the first trial values are then made and entered in the A_P-columns and repeated with the aid of rubber and pencil until all resultant $-\theta_{ik}$ values are negative so that θ_{ik} is positive (a very small negative value may be ignored). Adjustments quickly achieve the desired result when made according to the following rules :—

(*a*) Adjust resultant rotations in order of magnitude of error starting with the largest.

(b) Make the adjustment to the total rotation at each hinge by adjusting the assumed bending-moment at that hinge, and so cause least disturbance to other hinge rotation values.

(*c*) Repeat (*a*) and (*b*) for each hinge rotation requiring adjustment until the desired degree of proximity to positive θ_{ik} values is obtained (that is, negative $-\theta_{ik}$ values).

(*d*) Avoid, so far as is possible, obtaining final plastic-moment values which give an uneconomic distribution of bending moments.

It is generally found that beam hinges, except the end ones and inside column hinges, require little or no adjustment. Outside beam and column hinges often require adjustment, since suitable values depend upon the restraint given by the outside columns to the end beams and the relation of the value of M to m. Occasionally, if a wrong assessment of the positions of maximum bending moment for elastic conditions has been made, it may be necessary to alter a hinge position or vary an $\frac{l}{EI}$ value in order to obtain suitable θ_{ik} values at all hinges. The method given by Neal and Symonds [3] for locating plastic hinges may be useful in such cases.

The final θ_{ik} values must also be checked for excessive rotation which may not develop without the concrete crushing. Such cases may occur in regard to the support hinges of the beams. Rotations can then be reduced by increasing the assumed plastic-moment values at the hinges concerned.

The resultant bending-moment distribution is obtained by adding together the bending moments for the various cases, *Figs 6* (*b*) to (*l*), assuming the final plastic moment values to act at the hinges. These are indicated in *Figs 8* (*a*). The framework must be reinforced at all sections between plastic hinges so that these sections remain in an elastic condition when ultimate load is applied.

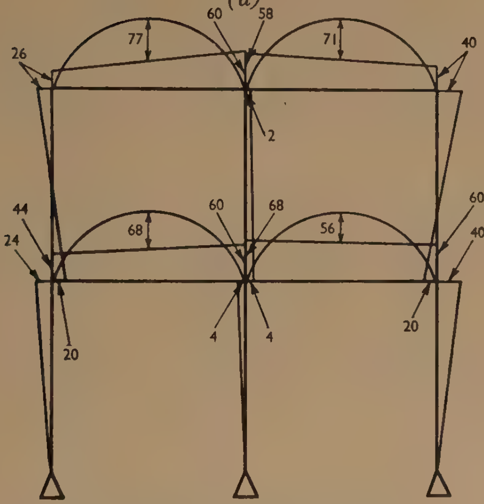
Elastic Condition at Hinge Points

It is only necessary to obtain an approximate solution for the distribution of bending moments for the fully elastic or working load condition, since the factor of safety of the structure depends upon the ultimate load, and can be calculated more precisely by the plastic-hinge than the elastic theory, but it may be necessary in some cases, for instance, in regard to the plastic-hinge moments at the supports of beams, to make an increase in plastic-moment value so that under working load, excessive strains or deflexions do not occur. A procedure of trial and adjustment similar to that already explained for the plastic condition of hinges can be used. Values of the plastic-hinge moments must be found by trial and adjustment which give approximately zero resultant rotation at each hinge point. Columns “ A_E ” in Table 1 show the adjustments to final plastic-moment values required to obtain the full elastic condition, and the corresponding rotation adjustments at each hinge which are added up and give approximately zero resultant rotation when added to the final plastic-rotation values. It will be seen that the main alteration in bending moments is at the supports, and that the variation to the column moments is small and will generally be of negligible effect, since the cross-section will be largely determined by the value of the direct load. The approximate elastic distribution of bending moments readily indicates whether or not the stresses at the supports will be excessive under working load. These are shown for the example in *Fig. 8* (*b*).

The trial-and-adjustment procedure for obtaining the approximate

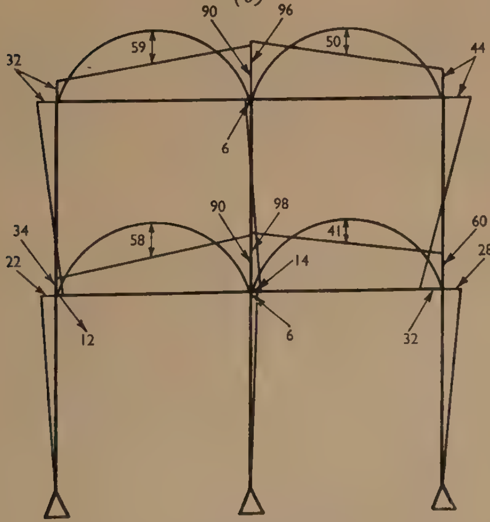
Figs 8

(a)



BENDING MOMENTS: PLASTIC CONDITION

(b)



BENDING MOMENTS: ELASTIC CONDITION

PLASTIC AND ELASTIC BENDING MOMENTS FOR FRAME EXAMPLE

distribution of bending moments for the elastic condition could, if required, be repeated until precise results were obtained. The plastic-hinge-moment values may then be denoted by unknowns X_1, X_2, etc., and the hinge rotations for unit moment acting as the δ_{ik} terms of the well known general elastic equations [1, 2, 4] which are obtained when the sum of the rotations at each hinge point is equated to zero. The trial-and-adjustment procedure recommended therefore provides a method of solution for such elastic equations.

Partial Elastic Condition

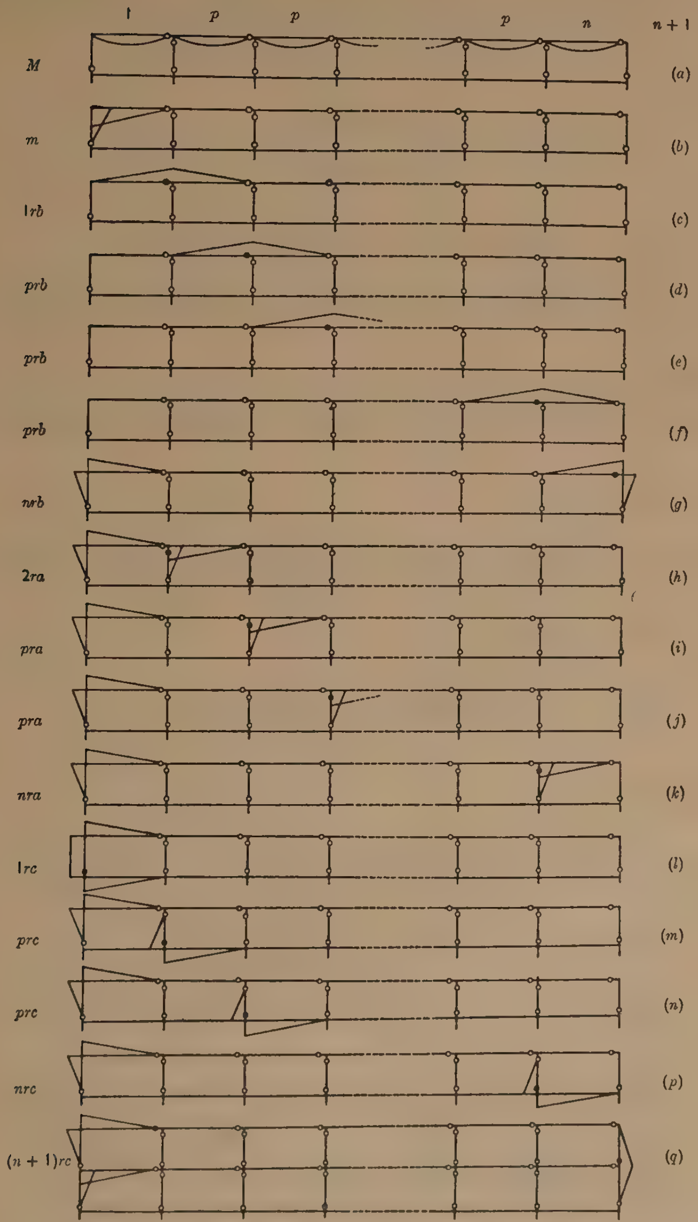
In some frameworks, certain members may be lightly loaded and therefore slender relative to the adjacent members, so that plastic hinges would not form in that member until excessive rotations had occurred in the stiffer members. In such a framework, the strains under working load in the stiffer members would tend to be excessive. To avoid such a condition, the slender members should either be stiffened or assumed to remain elastic under ultimate load. Thus, referring to the framework shown in *Figs 6*, if the $\frac{EJ}{h}$ value for the top right-hand column was reduced to one-quarter of the value which has been assumed, it can be seen from Table 1 that $-\theta_{ik}$ values with regard to hinges $2-2-b$ and $3-2-c$ would be greatly increased positively. Reduction of the plastic-moment values at those hinges would partly counteract this, but hinge $2-1-b$ would then have a reduced negative rotation value when acted on by $3-2-c$. Hinge $3-2-c$ might therefore preferably be considered to remain elastic, and hinge-moment values adjusted to give that hinge zero resultant rotation. Similar assumptions might be made in some frameworks having several slender members, and adjustments made until the θ_{ik} values were approximately zero.

General Expressions for Rotations of Plastic Hinges

For frameworks having a large number of storeys or bays, it is convenient to derive and use general expressions for the rotations of plastic hinges. Thus, referring to *Figs 9* which shows at (*a*) (*b*) . . . (*q*) the *r*th storey of a framework having *n* bays, general expressions have been derived so that by substituting bending-moment and stiffness values in them, the rotations of any hinge in a framework having any number of storeys or bays may be found. The expressions which are shown in Tables 2 (*a*) to (*d*) (pp. 288–291) have been derived for a suitable arrangement of plastic hinges for a framework subject to a combination of vertical load and horizontal load applied from the left.

They can be adapted for horizontal load applied from the right. The expressions have been derived by following the procedure already explained for the example shown in *Figs 6*, and can be checked by reference to *Figs 9* which indicate a typical storey of a building having any number of storeys,

Figs 9 (a) to (q)



HINGES AND PLASTIC MOMENTS FOR A MULTI-STOREY MULTI-BAY BUILDING FRAME

that is, the rth storey with typical end and internal bays. It will be seen that $r-1$ terms are zero for the bottom storey and $r+1$ terms for the top storey, otherwise the expressions repeat for each storey.

TABLE 2 (*a*)

BEAMS (*b*)

$$-\theta_{prb}=\frac{l_{pr}}{EI_{pr}}\left(-\frac{M_{pr}}{3}+\frac{X_{(p-1)rb}}{6}+\frac{X_{prb}}{3}-\frac{X_{pra}}{6}-\frac{X_{p(r+1)c}}{6}\right)$$
$$+\frac{l_{(p+1)r}}{EI_{(p+1)r}}\left(-\frac{M_{(p+1)r}}{3}+\frac{X_{prb}}{3}+\frac{X_{(p+1)rb}}{6}-\frac{X_{(p+1)ra}}{3}-\frac{X_{(p+1)(r+1)c}}{3}\right)$$

Repeat for P $= 2$ to $P = n - 1$

Repeat for all other storeys; in the top storey the $(r + 1)$ terms $= 0$

Special Case: P $= 1$

$$-\theta_{1rb}=\frac{l_{1r}}{EI_{1r}}\left(-\frac{M_{1r}}{3}-\frac{m_r}{6}+\frac{X_{1rb}}{3}+\frac{X_{nrb}}{6}+\sum_{\bar{x}_2}^{\bar{x}_n}\frac{X_{pra}}{6}+\sum_{\bar{x}_1}^{\bar{x}_{n+1}}\frac{X_{prc}}{6}-\frac{X_{1(r+1)c}}{6}\right.$$
$$\left.-\frac{X_{(n+1)(r+1)c}}{6}\right)$$
$$+\frac{l_{2r}}{EI_{2r}}\left(-\frac{M_{2r}}{3}+\frac{X_{1rb}}{3}+\frac{X_{2rb}}{6}-\frac{X_{2ra}}{3}-\frac{X_{2(r+1)c}}{3}\right)$$

Special Case: $P = n$

$$-\theta_{nrb}=\frac{l_{1r}}{EI_{1r}}\left(-\frac{M_{1r}}{3}-\frac{m_r}{3}+\frac{X_{1rb}}{6}+\frac{X_{nrb}}{3}+\sum_{\bar{x}_2}^{\bar{x}_n}\frac{X_{pra}}{3}+\sum_{\bar{x}_1}^{\bar{x}_{n+1}}\frac{X_{prc}}{3}\right.$$
$$\left.-\frac{X_{1(r+1)c}}{3}-\frac{X_{(n+1)(r+1)c}}{3}\right)$$
$$+\frac{l_{nr}}{EI_{nr}}\left(-\frac{M_{nr}}{3}+\frac{X_{(n-1)rb}}{6}+\frac{X_{nrb}}{3}-\frac{X_{nra}}{6}-\frac{X_{n(r+1)c}}{6}\right)$$
$$+\frac{h_{1r}}{EJ_{1r}}\left(-\frac{m_r}{3}+\frac{X_{nrb}}{3}+\sum_{\bar{x}_2}^{\bar{x}_n}\frac{X_{pra}}{3}+\frac{X_{1rc}}{2}+\sum_{\bar{x}_2}^{\bar{x}_{n+1}}\frac{X_{prc}}{3}-\frac{X_{(n+1)(r+1)c}}{3}\right)$$
$$+\frac{h_{(n+1)r}}{EJ_{(n+1)r}}\left(\frac{X_{nrb}}{3}-\frac{X_{(n+1)rc}}{6}-\frac{X_{(n+1)(r+1)c}}{3}\right)$$

TABLE 2 (*b*)

COLUMNS (*a*)

$$-\theta_{pra} = \frac{l_{1r}}{EI_{1r}}\left(-\frac{M_{1r}}{3} - \frac{m_r}{3} + \frac{X_{1rb}}{6} + \frac{X_{nrb}}{3} + \sum_{\bar{x}_2}^{\bar{x}_n}\frac{X_{pra}}{3} + \sum_{\bar{x}_1}^{\bar{x}_{n+1}}\frac{X_{prc}}{3}\right.$$
$$\left. - \frac{X_{1(r+1)c}}{3} - \frac{X_{(n+1)(r+1)c}}{3}\right)$$
$$+ \frac{l_{pr}}{EI_{pr}}\left(\frac{M_{pr}}{3} - \frac{X_{(p-1)rb}}{3} - \frac{X_{prb}}{6} + \frac{X_{pra}}{3} + \frac{X_{p(r+1)c}}{3}\right)$$
$$+ \frac{h_{1r}}{EJ_{1r}}\left(-\frac{m_r}{3} + \frac{X_{nrb}}{3} + \sum_{\bar{x}_2}^{\bar{x}_n}\frac{X_{pra}}{3} + \frac{X_{1rc}}{2} + \sum_{\bar{x}_2}^{\bar{x}_{n+1}}\frac{X_{prc}}{3} - \frac{X_{(n+1)(r+1)c}}{3}\right)$$
$$+ \frac{h_{pr}}{EJ_{pr}}\left(\frac{X_{pra}}{3} - \frac{X_{prc}}{6}\right)$$

Top storey $(r + 1)$ terms $= 0$ There is no θ_{1a}

Special Case: $p = n$

$$-\theta_{nra} = \frac{l_{1r}}{EI_{1r}}\left(-\frac{M_{1r}}{3} - \frac{m_r}{3} + \frac{X_{1rb}}{6} + \frac{X_{nrb}}{3} + \sum_{\bar{x}_2}^{\bar{x}_n}\frac{X_{pra}}{3} + \sum_{\bar{x}_1}^{\bar{x}_{n+1}}\frac{X_{prc}}{3}\right.$$
$$\left. - \frac{X_{1(r+1)c}}{3} - \frac{X_{(n+1)(r+1)c}}{3}\right)$$
$$+ \frac{l_{nr}}{EI_{nr}}\left(\frac{M_{nr}}{3} - \frac{X_{(n-1)rb}}{3} - \frac{X_{nrb}}{6} + \frac{X_{nra}}{3} + \frac{X_{n(r+1)c}}{3}\right)$$
$$+ \frac{h_{1r}}{EJ_{1r}}\left(-\frac{m_r}{3} + \frac{X_{nrb}}{3} + \sum_{\bar{x}_2}^{\bar{x}_n}\frac{X_{pra}}{3} + \frac{X_{1rc}}{2} + \sum_{\bar{x}_2}^{\bar{x}_{n+1}}\frac{X_{prc}}{3} - \frac{X_{(n+1)(r+1)c}}{3}\right)$$
$$+ \frac{h_{nr}}{EJ_{nr}}\left(\frac{X_{nra}}{3} - \frac{X_{nrc}}{6}\right)$$

Top storey $(r + 1)$ terms $= 0$

TABLE 2 (c)

COLUMNS (c)

$$-\theta_{1rc} = \frac{l_{1r}}{EI_{1r}}\left(-\frac{M_{1r}}{3} - \frac{m_r}{3} + \frac{X_{1rb}}{6} + \frac{X_{nrb}}{3} + \sum_{\bar{x}_2}^{\bar{x}_n}\frac{X_{pra}}{3} + \sum_{\bar{x}_1}^{\bar{x}_{n+1}}\frac{X_{prc}}{3} - \frac{X_{1(r+1)c}}{3}\right.$$
$$\left. - \frac{X_{(n+1)(r+1)c}}{3}\right)$$
$$+ \frac{l_{1(r-1)}}{EI_{1(r-1)}}\left(+\frac{M_{1(r-1)}}{3} + \frac{m_{(r-1)}}{3} - \frac{X_{1(r-1)b}}{6} - \frac{X_{n(r-1)b}}{3} - \sum_{\bar{x}_2}^{\bar{x}_n}\frac{X_{p(r-1)a}}{3}\right.$$
$$\left. + \frac{X_{1rc}}{3} - \sum_{\bar{x}_1}^{\bar{x}_{n+1}}\frac{X_{p(r-1)c}}{3} + \frac{X_{(n+1)rc}}{3}\right)$$
$$+ \frac{h_{1r}}{EJ_{1r}}\left(-\frac{m_r}{2} + \frac{X_{nrb}}{2} + \sum_{\bar{x}_2}^{\bar{x}_n}\frac{X_{pra}}{2} + X_{1rc} + \sum_{\bar{x}_2}^{\bar{x}_{n+1}}\frac{X_{prc}}{2} - \frac{X_{(n+1)(r+1)c}}{2}\right)$$

$(r + 1)$ terms $= 0$ where r is top storey

$(r - 1)$ terms $= 0$ where r is bottom storey

$$-\theta_{prc} = \frac{l_{1r}}{EI_{1r}}\left(-\frac{M_{1r}}{3} - \frac{m_r}{3} + \frac{X_{1rb}}{6} + \frac{X_{nrb}}{3} + \sum_{\bar{x}_2}^{\bar{x}_n}\frac{X_{pra}}{3} + \sum_{\bar{x}_1}^{\bar{x}_{n+1}}\frac{X_{prc}}{3}\right.$$
$$\left. - \frac{X_{1(r+1)c}}{3} - \frac{X_{(n+1)(r+1)c}}{3}\right)$$
$$+ \frac{l_{p(r-1)}}{EI_{p(r-1)}}\left(\frac{M_{p(r-1)}}{3} - \frac{X_{(p-1)(r-1)b}}{3} - \frac{X_{p(r-1)b}}{6} + \frac{X_{p(r-1)a}}{3} + \frac{X_{prc}}{3}\right)$$
$$+ \frac{h_{1r}}{EJ_{1r}}\left(-\frac{m_r}{3} + \frac{X_{nrb}}{3} + \sum_{\bar{x}_2}^{\bar{x}_n}\frac{X_{pra}}{3} + \frac{X_{1rc}}{2} + \sum_{\bar{x}_2}^{\bar{x}_{n+1}}\frac{X_{prc}}{3} - \frac{X_{(n+1)(r+1)c}}{3}\right)$$
$$+ \frac{h_{pr}}{EJ_{pr}}\left(-\frac{X_{pra}}{6} + \frac{X_{prc}}{3}\right)$$

$(r + 1)$ terms $= 0$ where r is top storey

$(r - 1)$ terms $= 0$ where r is bottom storey

The equation for $-\theta_{nrc}$ is similar, substituting n for p

TABLE 2 (d)

COLUMNS (c)

$$
\begin{aligned}
-\theta_{(n+1)rc} = {} & \frac{l_{1r}}{EI_{1r}}\left(-\frac{M_{1r}}{3}-\frac{m_r}{3}+\frac{X_{1rb}}{6}+\frac{X_{nrb}}{3}+\sum_{\bar{x}_2}^{\bar{x}_n}\frac{X_{pra}}{3}+\sum_{\bar{x}_1}^{\bar{x}_{n+1}}\frac{X_{prc}}{3}\right. \\
& \left.-\frac{X_{1(r+1)c}}{3}-\frac{X_{(n+1)(r+1)c}}{3}\right) \\
& +\frac{l_{1(r-1)}}{EI_{1(r-1)}}\left(\frac{M_{1(r-1)}}{3}+\frac{m_{(r-1)}}{3}-\frac{X_{1(r-1)b}}{6}-\frac{X_{n(r-1)b}}{3}-\sum_{\bar{x}_2}^{\bar{x}_n}\frac{X_{p(r-1)a}}{3}\right. \\
& \left.-\sum_{\bar{x}_1}^{\bar{x}_{n+1}}\frac{X_{p(r-1)c}}{3}+\frac{X_{1rc}}{3}+\frac{X_{(n+1)rc}}{3}\right) \\
& +\frac{h_{1r}}{EJ_{1r}}\left(-\frac{m_r}{3}+\frac{X_{nrb}}{3}+\sum_{\bar{x}_2}^{\bar{x}_n}\frac{X_{pra}}{3}+\frac{X_{1rc}}{2}+\sum_{\bar{x}_2}^{\bar{x}_{n+1}}\frac{X_{prc}}{3}-\frac{X_{(n+1)(r+1)c}}{3}\right) \\
& +\frac{h_{1(r-1)}}{EJ_{1(r-1)}}\left(\frac{m_{(r-1)}}{3}-\frac{X_{n(r-1)b}}{3}-\sum_{\bar{x}_2}^{\bar{x}_n}\frac{X_{p(r-1)a}}{3}-\frac{X_{1(r-1)c}}{2}-\sum_{\bar{x}_2}^{\bar{x}_{n+1}}\frac{X_{p(r-1)c}}{3}\right. \\
& \left.+\frac{X_{(n+1)rc}}{3}\right) \\
& +\frac{h_{(n+1)r}}{EJ_{(n+1)r}}\left(-\frac{X_{nrb}}{6}+\frac{X_{(n+1)rc}}{3}+\frac{X_{(n+1)(r+1)c}}{6}\right) \\
& +\frac{h_{(n+1)(r-1)}}{EJ_{(n+1)(r-1)}}\left(-\frac{X_{n(r-1)b}}{3}+\frac{X_{(n+1)rc}}{3}+\frac{X_{(n+1)(r-1)c}}{6}\right)
\end{aligned}
$$

Use of Blocks and Critical Hinges

Critical plastic hinges of a framework such as is shown in *Fig. 2* can be selected so that the general expressions need only be applied to determine the rotations of a few hinges in blocks such as those marked 1 to 5 in which each of the storeys are identical. The calculations for the rotations of similar hinges in other blocks will generally be the same except for adjustment to the M, I, and $\frac{h}{J}$ values. The work of solving a framework which is many hundreds of times statically indeterminate can thus be reduced to checking and adjusting the rotations of a few hinges in one block and making some further adjustments for the corresponding hinges in other blocks.

The rth storey of one of the blocks of the framework shown in *Fig. 2* may be used to demonstrate the application of the general expression.

The following values are given :— $\frac{l}{EI} = 12$ in all beams

$$\frac{h}{EJ} = 6 \text{ in all columns}$$

$$M = 10 \text{ in all beams}$$

$$m_r = 1$$

$$m_{r-1} = 1{\cdot}1$$

As a first trial, assume the following values :—

$$\bar{X}_{1rb} = \bar{X}_{prb} = \bar{X}_{1r-1b} = \bar{X}_{pr-1b} = X_{1r+1b} = X_{pr+1b} = \frac{M}{2} = 5$$

$$\bar{X}_{1rc} = \bar{X}_{1r-1c} = \bar{X}_{n+1r-1c} = \bar{X}_{n+1rc} = \frac{0{\cdot}67M \times 2}{2+2+1}$$

$$= 2{\cdot}68, \text{ say } 2{\cdot}5$$

$$\bar{X}_{nr-1b} = \bar{X}_{nrb} = 2 \times 2{\cdot}68 + \frac{1 \times 2}{10} = 5{\cdot}56, \text{ say } 4{\cdot}0$$

$$\bar{X}_{prc} = \bar{X}_{pra} = \bar{X}_{nrc} = X_{nra} = \frac{m_r}{10} = 0{\cdot}1$$

$$\bar{X}_{1r+1a} = \bar{X}_{n+1r+1a} = 2{\cdot}4$$

$$\bar{X}_{pr+1a} = 0{\cdot}08$$

$$\bar{X}_{pr-1a} = 0{\cdot}1$$

The plastic-hinge values for the $r+1$th storey have been assumed, but in a fully worked out example would have been already determined for the block above. The values for the $r-1$th-storey hinges, when the rth storey is at the bottom of a block must be assumed, and if greatly varied when the block below is investigated, the influence of the variation on the rth-storey rotations, which will generally be small, should be checked.

Applying the general expressions in Table 2 for the rotations of the twelve critical hinges of the rth storey, first-trial θ_{ik} values are obtained as shown at the bottom of Table 3 (facing p. 281). The most erroneous assumption is seen to have been made in regard to $1-r-c$ due to the restraint moment of the outside column in regard to the end beam being greater in relation to the sway moment. The adjustments to the rotations are indicated in the A_P columns, and they give satisfactory final-trial resultant-rotation values for all the hinge points as shown at the bottom of the Table. Since all moment values and dimensions remain the same for each storey in the block, except m_r values which increase down the building, the top-storey hinges will be critical for minimum values of rotations and the

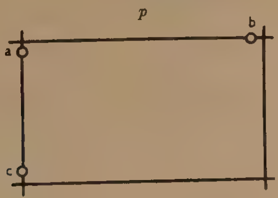
bottom for maximum values in regard to beam hinges, since m_r terms do not occur in the general expression for $-\theta_{prb}$, and are negative in value in the expression for $-\theta_{1rb} - \theta_{nrb}$. Also in the same expressions $(r + 1)$ terms are all negative, so that if the rth storey is the top storey of a block, their value will be less than for any other storey, since the plastic-hinge moments of the block above will be less than those for the corresponding hinges in the block under consideration. Generally it will be evident that rotation of corresponding top- and bottom-storey hinges in a block differ by only small amounts, so that the hinges of only one storey in each block need be investigated. The column-hinge rotations will generally be small and obviously not excessive, and the top storey is likely to be critical for minimum values, but a definite rule cannot be stated since $(r + 1)$ terms of both negative and positive sign occur in the general expressions for column-hinge rotations. Generally, an examination of one storey is sufficient, but in some cases it may be necessary to check both top and bottom storeys of a block to ensure that in both cases :

(*a*) all resultant rotations are positive in value but not excessive ; and

(*b*) approximate bending moments for the elastic condition are not excessive,

the latter check being carried out as already explained.

Figs 9 (r)



HINGES AND PLASTIC MOMENTS FOR THE rTH STOREY OF A BUILDING FRAME HAVING n BAYS

Adjustments for the elastic condition are shown in the " A_E " columns of Table 3. The beam hinges are mainly affected. The final-trial total θ_{ik} values are reduced to approximately zero by the total " A_E " values.

The resultant bending moments at the hinges will be the original assumed values plus the final adjustment values, and between-hinge values may be obtained by superimposing the bending-moment diagrams in *Figs 9*.

It is evident that in frameworks supporting uniformly distributed load and having uniform beam and column sections, and approximately equal beam spans and column heights, approximately the following ultimate-bending moments develop :

(1) At mid-span and internal supports of beams : $\frac{wl^2}{16}$ for vertical load.

(2) Column and beam sway moments calculated from assuming points of contra-flexure to be at the mid-height of each storey and mid-span of each beam, and wind shear divided between the columns in proportion to their stiffness.

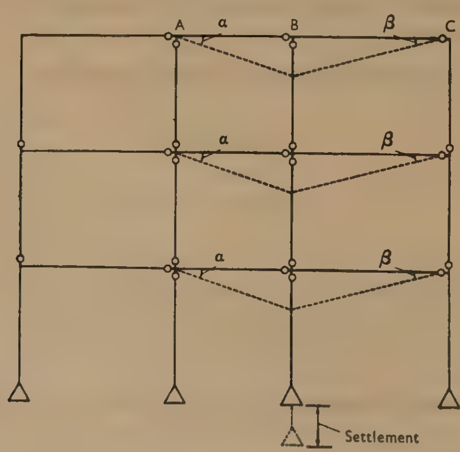
(3) Outside-column moments approximately as obtained by (2) plus restraint moments calculated by distributing the end "fixed" moment of the end beam for vertical load among the beam and upper and lower columns in proportion to their stiffness.

(4) End-beam support moments equal to the sum of the adjacent column restraint moments, calculated as in (3), plus sway moments calculated as in (2).

Effect of Settlement

Referring to *Fig. 10*, it can be seen that settlement of one column relative to another does not affect the distribution of bending moments, but only

Fig. 10



INFLUENCE OF SETTLEMENT ON HINGE ROTATIONS

increases the angle of rotation of certain hinges. Thus the slope of the ends of the beams adjacent to columns A and C increases α and β and decreases $\alpha + \beta$ adjacent to column B. The rotations of the hinges at these points can therefore be adjusted accordingly, the value of $\alpha + \beta$ depending only upon the relative settlement of the columns and the dimensions of the beams.

Conclusions

The plastic-hinge method of design enables frameworks many hundreds of times statically indeterminate to be analysed. The true factor of safety

and the stresses under working load can be determined with sufficient accuracy to ensure that excessive strains will not occur. The method could be used in practice by designers who have experience of elastic theory, and inexperienced designers could carry out much of the work under the direction of suitably qualified men. The method is realistic because it is based upon the real characteristics and behaviour of reinforced-concrete frameworks, and gives more accurate values of ultimate strength than the elastic theory. It justifies ignoring, in many cases, the effect of relative settlement which is not possible by elastic theory. Within fairly broad limits of stiffness ratios, which it should now be possible to compute, building frames could be designed by use of simple formulae and rules, such as (1) to (4) above.

Practical designers may claim that these values are the same as those in use at present, except for reductions in regard to beams in bending moments. This is partly true, but designers who have been using the mid-span-point-of-contraflexure rule for sway moments, regardless of the relative stiffness of members of the framework, have been fortunate in their intuitive judgement in many cases. Elastic theory, had they applied it strictly, would have proved their assumptions to have been wrong, whereas the plastic-hinge theory often would have proved them to have been right.

Safe Limiting Values of Rotations of Plastic Hinges

(See Tables 4 and 5, pp. 300–302, and *Fig. 11*)

Plastic hinges may be classified as follows :—

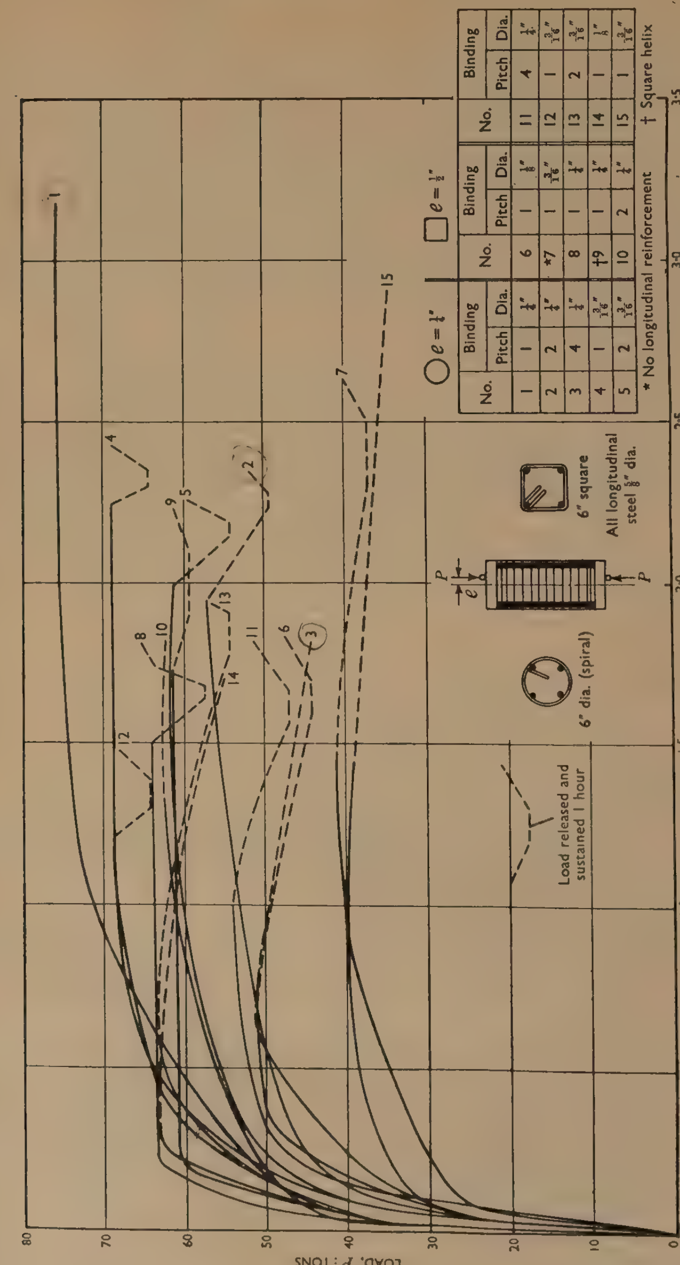
(1) *Tensile.*—Those occurring in beams or columns subjected to bending tension and reinforced with mild steel or steel which develops enough yield to avoid fracture before failure occurs by the concrete crushing.

(2) *Unbound compressive.*—Those occurring in beams or columns in which initial failure occurs by the concrete crushing, and in which effective binding is not used, so that plastic rotation is small.

(3) *Bound compressive.*—Those occurring in beams or columns in which the steel yield is small, or initial failure occurs by the concrete crushing, but in which effective binding is used locally so that large compressive strains or deformations can develop before failure occurs and without the section weakening as shown in *Figs 11* and *12*.

It has already been shown that, for tensile hinges, $\theta = \dfrac{s_p l_p}{n_1 d}$ (see p. 270 in reference 1). Such hinges form whenever yield begins to develop in the

Fig. 11



No.	Binding Pitch	Binding Dia.	No.	Binding Pitch	Binding Dia.	No.	Binding Pitch	Binding Dia.
1	1	¼"	6	1	⅛"	11	4	¼"
2	2	¼"	*7	1	3/16"	12	1	3/16"
3	4	¼"	8	1	¼"	13	2	3/16"
4	1	3/16"	†9	1	¼"	14	1	⅛"
5	2	3/16"	10	2	¼"	15	1	3/16"

STRESS/STRAIN CHARACTERISTICS OF CONCRETE PRISMS BOUND BY CLOSE-SPACED LINKS

Fig. 12

Test Specimen having the Stress/Strain Characteristics shown in *Fig.* 11 under Eccentric Load

Fig. 17

Distribution of Cracks prior to Bond Slip in Beam Tested at Imperial College by Ziauddin

Fig. 20

Reinforced-Concrete Shell Tested at Imperial College

Fig. 21

Reinforced-Concrete Shell after Failure in Compression accompanied by Buckling

tensile reinforcement. Local slip widens the cracks, and their distribution indicates that generally it is safe to assume that s_p is constant over l_p, and that it is safe to assume that l_p equals the effective depth of the beam. s_p may be assumed to have a safe limiting value of 0·002 unless special close binding is provided. $n_1 = \frac{T}{\alpha bc'd}$ (p. 271 in reference 1) for safe limiting values (see also Table 5).

For compression hinges of both types, $\theta = \frac{l_{psd}}{d}$ (p. 273 in reference 1). In columns, plastic-rotation values can often be made to be small without uneconomic distributions of bending moments occurring, so that safe limiting values of $s_d = 0{\cdot}001^2$ (*Fig. 5*) and $l_p = \frac{d}{2} +$ half beam depth, which are appropriate for unbound compressive hinges, give sufficiently great values of θ. In highly reinforced beams and in columns when the value of θ is high, special binding may be used as indicated by *Figs 11* and *12*. s_d may then be assumed to have a safe limiting value of 0·01.

The value of l_p should not be assumed greater than d or one-tenth of the length of the column or beam, since each hinge is assumed to be concentrated at a point, and the possible spreading of the peak of the idealized bending-moment diagram is limited. The value of I assumed in calculating the deformations of the adjoining members must be low enough to allow for slight plasticity or bond slip at other sections of high stress, although sufficient reinforcement is provided at such sections to ensure that a nominal elastic condition is maintained.

Fig. 11 indicates the stress-strain characteristics of a number of stirrup-bound concrete prisms of the type shown in *Fig. 12* and tested by Chan.[5] The use of closed stirrups is evidently an effective practical way of increasing the available compressive strain over a short length of reinforced concrete so that a satisfactory compressive hinge can form.

The following indicates the length of plastic hinge required for typical values of θ.

In a beam having close-bound stirrups, a high value of θ would be $\frac{Ml}{3EI}$, so that if:

$$M = 0{\cdot}2\, bd^2C_u,\ E = 3 \times 10^6 \text{ and } I = \frac{bd^3}{15},\ C_u = 6 \times 10^3,\ s_p = 0{\cdot}01,$$

$$n_1 = 0{\cdot}25,\ l_p = xl.$$

then: $$\theta = \frac{S_pl_p}{n_1d} = \frac{Ml}{3EI}$$

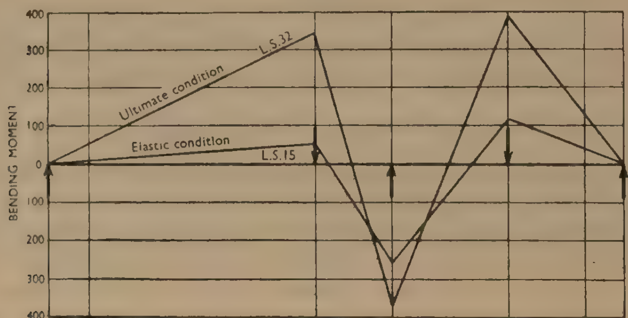
Substituting the above values, $x = 0{\cdot}05$.

It is evident therefore that for high values of θ, bound hinges of short

length are sufficiently plastic, and when θ is small, often unbound hinges of length, say, $\frac{\text{span}}{15}$, would provide sufficient rotation.

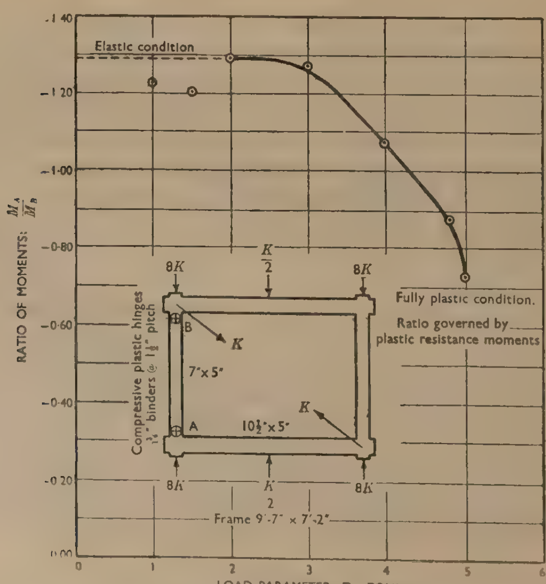
Fig. 13 shows the elastic ultimate distribution of bending moments in a beam continuous over three supports, which was tested at Imperial College by Everard [6]; see also *Fig. 30* and pp. 309, 310 in reference 1. Failure

Fig. 13



ELASTIC AND ULTIMATE DISTRIBUTION OF BENDING MOMENTS IN A CONTINUOUS BEAM

Fig. 14



ELASTIC AND ULTIMATE DISTRIBUTION OF BENDING MOMENTS IN A FRAME

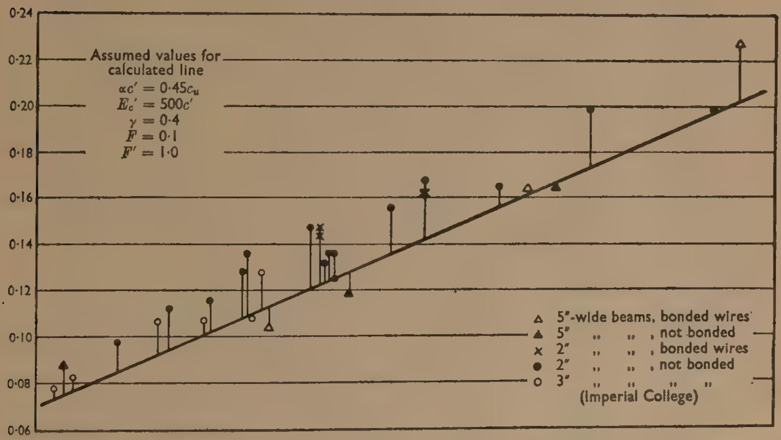
was initiated by steel yield and did not occur until full moment re-distribution had taken place. *Fig. 14* indicates redistribution of bending moments due to compressive plastic hinges in a 10-feet-by-8-feet frame tested by Moss-Morris [15] at Imperial College.

ULTIMATE STRENGTH OF PRESTRESSED BEAMS—EXPERIMENTAL RESULTS

(Applicable to plastic hinges)

The notation is given in reference 1, p. 275. Most of the experimental results have already been published,[7] and Table 4 indicates the scatter of the values of those factors governing ultimate strength which cannot be obtained from specimen tests.

Fig. 15



CALCULATED AND EXPERIMENTAL VALUES OF $\frac{R}{C_u}$ FOR OVER-REINFORCED PRESTRESSED BEAMS OF VARYING DEGREES OF PRESTRESS AND BOND

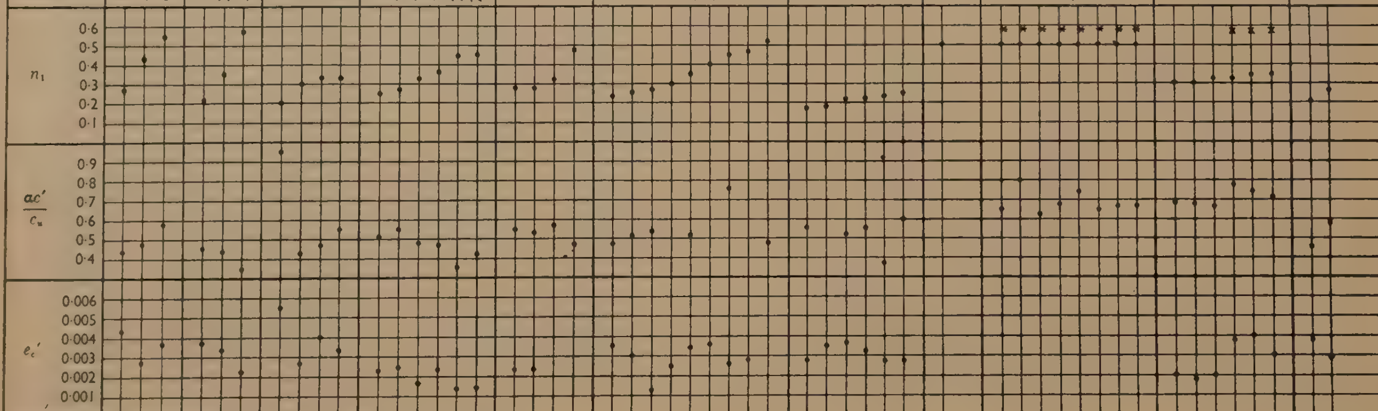
Fig. 15 compares experimental results obtained at Imperial College by the Department of Scientific and Industrial Research group and others with theoretical results based on the safe limiting values given. Some of the beams are certainly small, but the D.S.I.R. beams are large enough for reasonable accuracy and cover a wide range of neutral-axis depth. The results indicate that the safe limiting values proposed in Table 5 are reasonable.

Table 5 summarizes conclusions made from the results shown in Table 4.

The notation is given on p. 275 of reference 1, except for the following, which occur in *Fig. 34*, reference 1 :

TABLE 4.—VALUES OF FACTORS GOVERNING THE BENDING STRENGTH OF REINFORCED CONCRETE AND PRESTRESSED BEAMS—OBTAINED FROM BEAM TESTS

DETAILS OF BEAMS	Imperial College D.S.I.R. Group		Gifford (Imp. Coll.)	Ziauddin (Imp. Coll.)		Lao (Imp. Coll.)		King	R. H. Evans (Leeds U.)	Bates (B.R.S.)	Ziauddin (Imp. Coll.)
EFFECTIVE SIZE	5·0" × 5·2" (2); 4·9" × 8" (4, 6, 1, 3, 5)		2·7" × 4·5", 2·9" × 4·5", 2·5" × 4·5", 2·2" × 4·5"	2" × 2·8"		2" × 2·8"	3" × 3·2"	3" × 7"	2·5" × 8·1", 2·5" × 8·2", 2·5" × 8·2", 2·6" × 8·2", 2·5" × 8·1", 2·6" × 8·1", 2·5" × 8·1", 2·5" × 8·1"	2" × 5·5"	5·1" × 8·2", 5·1" × 7·2"
BOND	Grouted	Not bonded	Not bonded	Not bonded	Grouted	Not bonded	Not bonded	Not bonded		Bonded	Not b., Bonded
SERIAL NO.	2, 4, 6	1, 3, 5	3, 4, 2, 1	1, 25, 21, 24, 23, 22	1, 3, 2, 5	12, 34, 24, 33, 32, 31, 25, 30, 22	3, 11, 2, 13, 1, 4	1	1, 2, 10, 12, 14, 15, 16, 17	J, O, P, C, A, D	E2, B2



✱ Values of n_1 assumed from prestress and failure by crushing

× Values of n_1 from mean depth of all wires

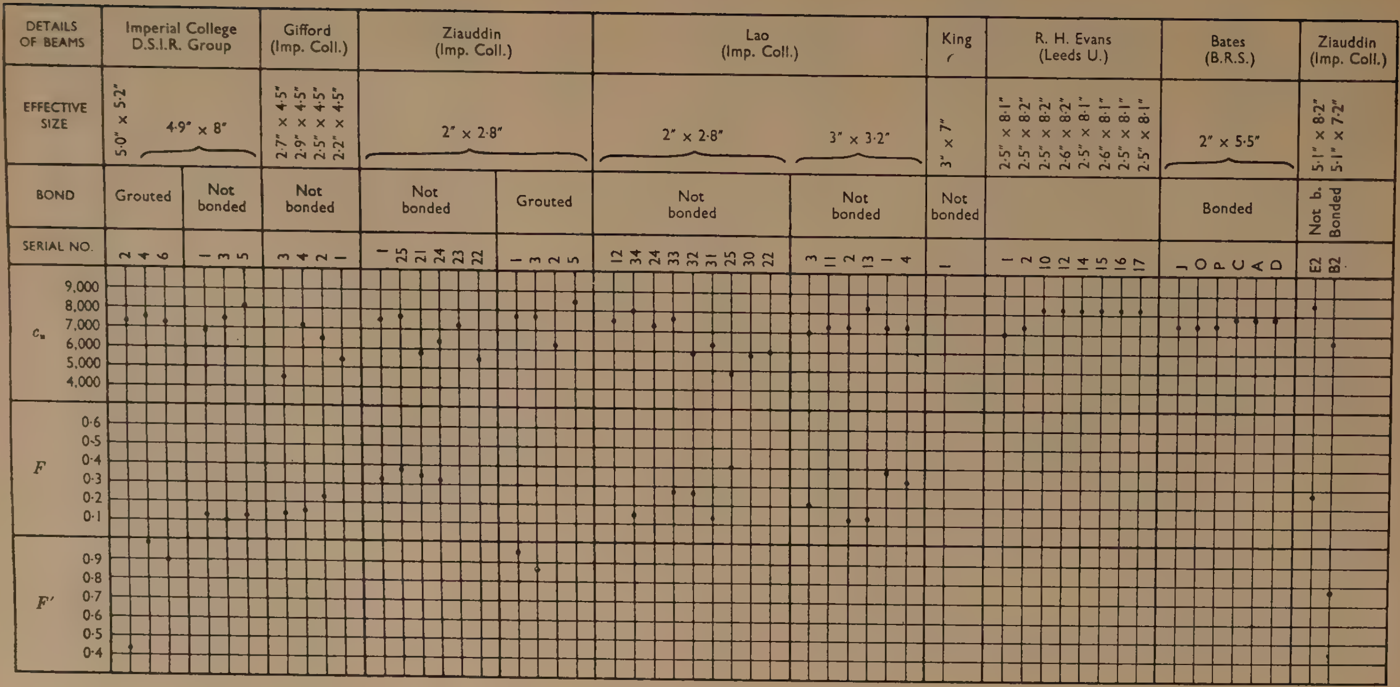

DETAILS OF BEAMS
Imperial College D.S.I.R. Group
Gifford (Imp. Coll.)
Ziauddin (Imp. Coll.)
Lao (Imp. Coll.)
King
R. H. Evans (Leeds U.)
Bates (B.R.S.)
Ziauddin (Imp. Coll.)
EFFECTIVE SIZE
5·0" × 5·2"
4·9" × 8"
2·7" × 4·5"
2·9" × 4·5"
2·5" × 4·5"
2·2" × 4·5"
2" × 2·8"
2" × 2·8"
3" × 3·2"
3" × 7"
2·5" × 8·1"
2·5" × 8·2"
2·5" × 8·2"
2·6" × 8·2"
2·5" × 8·1"
2·6" × 8·1"
2·5" × 8·1"
2·5" × 8·1"
2" × 5·5"
5·1" × 8·2"
5·1" × 7·2"
BOND
Grouted
Not bonded
Not bonded
Not bonded
Grouted
Not bonded
Not bonded
Not bonded
Bonded
Not b.
Bonded
SERIAL NO.
2 4 6
1 3 5
3 4 2 1
1 25 21 24 23 22
1 3 2 5
12 34 24 33 32 31 25 30 22
3 11 2 13 1 4
1
1 2 10 12 14 15 16 17
J O P C A D
E2 B2
9,000
8,000
7,000
6,000
5,000
4,000
c_u
0·6
0·5
0·4
0·3
0·2
0·1
F
0·9
0·8
0·7
0·6
0·5
0·4
F'

TABLE 5.—PROPOSED SAFE LIMITING VALUES

Factor	Safe limiting value	
$\alpha c'$	$0{\cdot}5C_u$	When C_u is greater than 4,000 lb. per square inch, so that if $c'=0{\cdot}75\ C_u$ then $\alpha=0{\cdot}67$.
	$0{\cdot}6C_u$	When C_u is less than 4,000 lb. per square inch. When a factory-standard of quality control is maintained, the value of C_u may be assumed to be the minimum works-cube test-strength multiplied by 0·8 to allow for strength deviations in the structure, and a load factor of safety of 2 used. Safe limiting values of e'_c and e'_s give a safe limiting position of the neutral axis, which will generally be lower, if the concrete is occasionally slightly weak. The assumption of a high value of C_u is therefore justified, since it is not likely that local weakness of the concrete and a small ultimate strain causing a high position of the neutral axis will occur simultaneously.
γ	0·4	
E'_c	$500c'$	
F'	1·0	When bars greater than ½-inch diameter are used, or bond is obtained by grouting, a value of 0·85 should be used.
F	0·1	Values range from 0·1 to 0·35; full bond should therefore always be provided by grouting unless failure is assumed to occur at the commencement of cracking.

Example.—Assuming the following values for a fully prestressed and bonded beam in which total ultimate tension (which may be assumed smaller than breaking stress to avoid high steel strain) equals total ultimate compression.

$$t_y = 200{,}000 \quad E'_s = 25 \times 10^6 \quad F' = 1$$
$$\beta = 0{\cdot}75 \quad \alpha c' = 0{\cdot}5C_u \quad \gamma = 0{\cdot}4 \quad E'_c = 500c'$$

Concrete compressive strain $= 0{\cdot}002$ virtual tensile strain $\doteq$

$$\frac{(1-0{\cdot}75)2 \times 10^5}{25 \times 10^6} = 0{\cdot}002$$

$$\therefore n' = 0{\cdot}5 \quad \therefore M' = \alpha b d^2 c'(n_1 - \gamma n_1^2) = 0{\cdot}2bd^2C_u.$$

T-*Beams and* I-*Beams.* The position of the neutral axis of a T-beam or an I-beam prior to failure can be determined from linear-strain distribution in the same way as rectangular beams, and the concrete moment of resistance by assuming a trapezoidal distribution of stress when the neutral axis is below the soffit of the flange, the ultimate stress being the prism strength or 65 to 80 per cent of the cube strength.

e'_s denotes change of strain of steel when the stress increases from βt_y to t_y.

e'_c ,, strain of concrete prior to failure at top edge of beam.

$F' = \dfrac{e'_s n_1}{e'_c(1-n_1)}$ in bonded beams, and corresponds to the strain factor F in end-anchored beams, which has been defined on

p. 275 of reference 1 and which, since average concrete strain = adjacent steel strain, also ≏

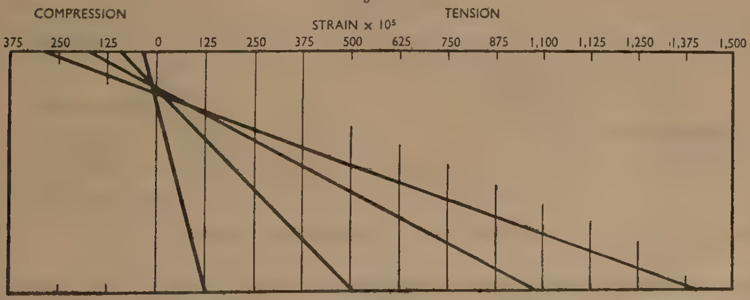
$$\frac{e'_s n_1}{e'_c(1 - n_1)} \text{ or } \frac{\dfrac{(1-\beta)t_y}{E'_s}}{\dfrac{c'}{E'_c} \times \dfrac{1-n_1}{n_1}}$$

In bonded beams, local bond slip causes wide cracking and hence raises the neutral-axis position, and so may reduce the value of F' below the normal value of 1.

The results shown in Table 4 indicate that there is no point in arguing about the precise safe limiting values of $\frac{\alpha c'}{C_u}$, γ, and E'_c. The governing factors in regard to ultimate strength, which are most uncertain, are values of C_u, F, F', and β, which depend on the standard of workmanship.

Fig. 16 indicates the concrete and steel strains measured over a length of 8 inches by Ziauddin at Imperial College, in May, 1952, in a 10-inch-by-5-inch bonded (by casting, not grouting) beam, reinforced with six 5 mm. non-prestressed high-tensile steel wires having an ultimate strength of 104

Fig. 16



Average Strain Distribution Measured on Gauge Length across Widest Crack. Ziauddin's Bonded Beam, B.2, 5·1 inches × 7·2 inches

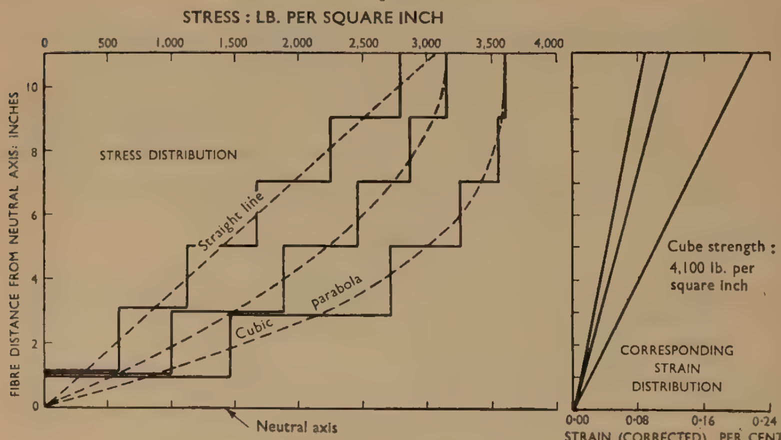
tons per square inch. Ultimate total bending compression would thus be about equal to ultimate total tension. The position of the neutral axis as indicated by the average measured strains, and which are shown in *Fig. 16*, must be very close to the position of the neutral axis prior to failure by bending. This is also indicated by the cracks shown in *Fig. 17* (facing p. 296). Failure actually occurred by sudden bond slip, but the ultimate bending strength had almost developed. If bond slip had not occurred, no appreciable lowering of the neutral axis could have taken place prior to failure. It is clear, therefore, contrary to Hajnal-Konyi's expectation (p. 121 of reference 7), that the position of the neutral axis, prior to failure, is governed mainly by the concrete and steel strains, and that when non-

prestressed high-tensile steel is used, the ultimate strength of beams in regard to failure caused by the concrete crushing is very much smaller than when mild steel of similar total strength is used, since before yield, the mild-steel strain is only about a quarter of the value of the high-tensile-steel strain and the neutral axis is at about half depth.

Bending-Simulation Machine—Experimental Results

R. W. Smithies[10] has obtained results which are of interest (see *Fig. 18*). Smithies assumed that in the elastic range, linear distributions of stress caused linear distributions of strain, and that errors based upon this assumption in regard to the strain readings in the concrete close to the

Fig. 18



Distribution of Compressive Stress

rams increased in proportion to the stress. *Fig. 18* shows his corrected readings. Research with the bending-simulation machine is continuing.

Ultimate Strength of Cylindrical Shells in Bending

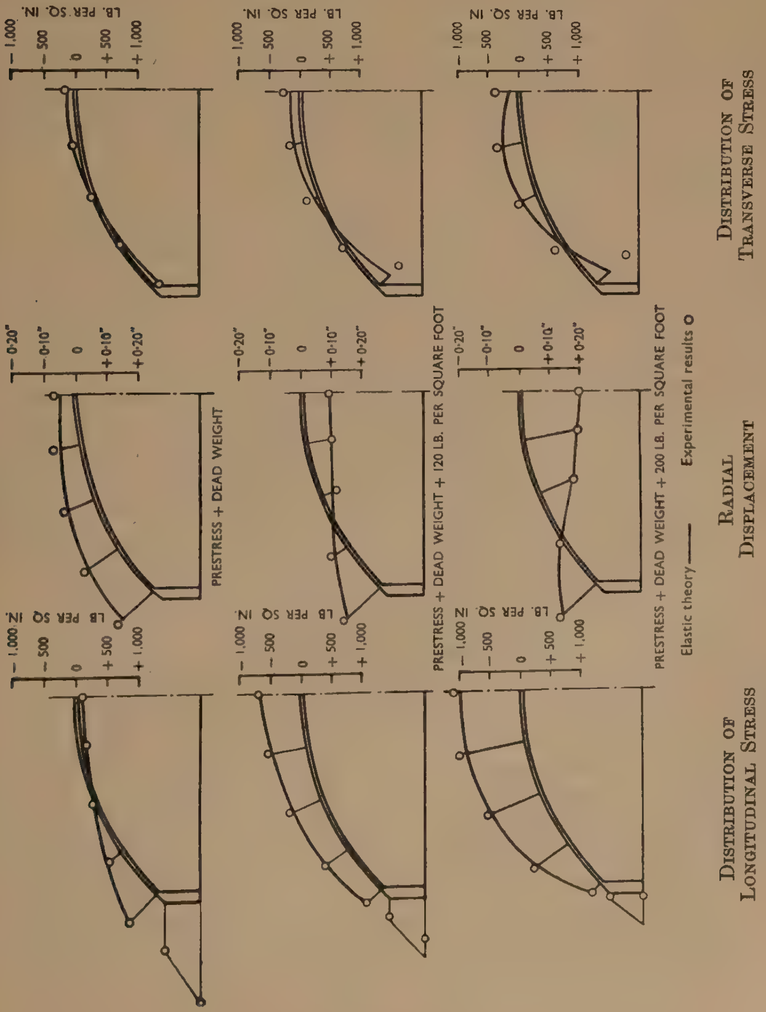
(See reference 1, p. 281)

Experimental Results

Using a steel-sheet model of a part-cylindrical shell, Selvanayagem[11] at Imperial College has determined the distribution of strain for various arrangements of load and edge restraint. His results show good agreement with the general elastic theory of Jenkins.

Shaker[12] has tested to destruction a prestressed concrete shell, having the following dimensions: 20 feet long, 8 feet wide, radius 5·8 feet, and ¾ inch thick. The shell was constructed by casting in turn sixteen segments in a single mould and joining them together by prestressed wires

passing longitudinally through the edges. The end diaphragms were cast in situ. Steel plate-dowels were used at the joints to provide some shear resistance, but these proved ineffective soon after the pre-compression stress across the joints was overcome by the compression stress due to load. Failure occurred under a load of about 220 lb. per square foot, and was initiated by the opening of the joint between the shell and the crown of the end diaphragms, due to loss of pre-compression at mid-span allowing slipping of the segments relative to each other. Thereafter the shell lost its shape, and collapse finally took place in transverse bending, when the shell edges became supported by the timber framework provided to break



Figs 19

the fall of the load. The distribution of direct longitudinal bending stress shown in *Figs. 19* is similar to that obtained by applying ordinary bending theory to a hollow beam. The values and distribution of the transverse stresses and radial displacements shown in *Figs 19* indicate that, in regard to transverse bending, the edge beam relative to the shell was very stiff vertically and in torsion. Transverse forces were membrane forces with small eccentricities, except close to the edge beam, which provided considerable restraint. The experimental values of stress obtained by Metzger gauges agree closely with stress calculations based on Schorer's assumptions, which neglect torsion in the shell and longitudinal bending, but take into account the deformations of the edge beams. No cracking, except for a hair crack at one joint at the bottom edge, occurred before failure, mainly because the shell consisted of pre-cast segments joined together by prestressed wires. While it was still in longitudinal compression throughout, the shell acted, as might be expected, as an uncracked elastic long shell with vertically stiff edge-beams, developing transverse stresses in accordance with elastic theory simplified by neglecting torsion and longitudinal bending.

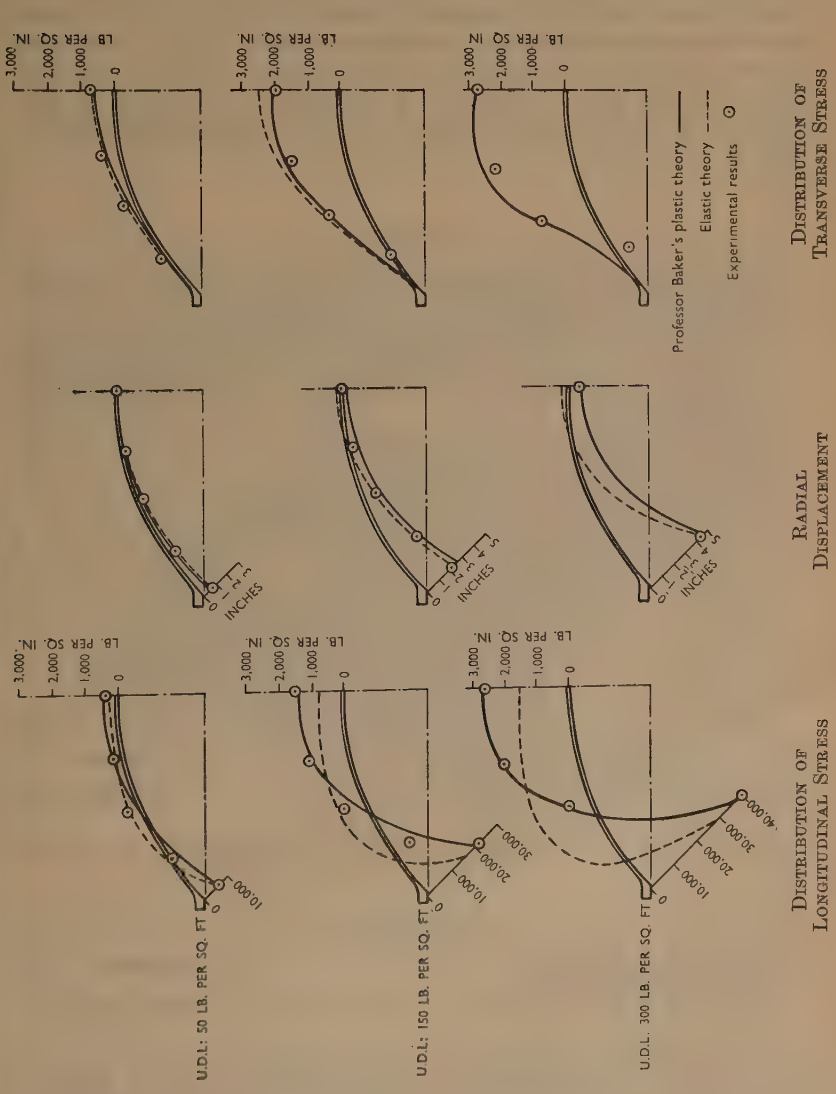
Gouda [13] tested to destruction (see *Figs 20* and *21*, forcing p. 297) a reinforced-concrete part-cylindrical shell, $\frac{7}{8}$ inch thick, without longitudinal edge beams, designed to fail by longitudinal compression. The prestressed shell constructed by Shaker was used as formwork, the dimensions being 21 feet 3 inches by 8 feet 6 inches wide, radius 6 feet, thickness $\frac{7}{8}$ inch, central angle 90 degrees. The values and distribution of stresses and deformations shown in *Figs 22* indicate that up to the stage of cracking, calculations based on the elastic theory using Schorer's approximations agreed closely with the experimental results. Prior to failure, tension cracks extended upwards and the distribution of longitudinal stress before failure agreed closely with the plastic theory.[1] Transverse bending stresses also agreed with the plastic theory when the upward vertical component of the main tensile edge-steel was taken into account, the edges prior to failure having sagged $4\frac{1}{2}$ inches at mid-span below the supports.

Collapse took place by failure of the concrete at the crown in longitudinal compression, accompanied by buckling at a concrete stress of about 3,000 lb. per square inch under a load of about 320 lb. per square foot. The concrete compression strength (cylinder) was about 4,000 lb. per square inch. The crown deflexion at mid-span was negligible until close to failure, when it increased with a small addition of load to about 1 inch, after which collapse occurred very suddenly. The stress in the main tensile steel was about 43,000 lb. per square inch. The wide cracks which occurred were probably caused by local bond slip, since the yield stress of the steel was 45,000 lb. per square inch, and cracks started at a load of only 100 lb. per square foot.

Full details of the above tests are recorded in theses by Selvanayagam,[11] Shaker,[12] and Gouda.[13]

Short Shells

The theory proposed in the first part of this Paper has now been extended to include short shells.[14]



Figs 22

Acknowledgements

The Author wishes to repeat the acknowledgements given in Structural Paper No. 26,[1] in particular the pioneer work of Professor J. F. Baker on the plastic theory, and to acknowledge the assistance of Yu Chan-Wah and Chan William Wai-Lee in preparing diagrams and checking the plastic-hinge theory and examples, and is also grateful for the access to the results of various experiments carried out by his students at Imperial College. Thanks are also due to Professor R. H. Evans, and Messrs J. W. H. King and S. C. C. Bate for their contributions to the results summarized in Table 4.

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16. Yu Chan-Wah, " A Study of the Plastic Hinge Theory." Ph.D. thesis (to be submitted).

The Paper is accompanied by six photographs and 23 sheets of diagrams, from which the half-tone page plates, the Figures in the text, and the folder between pp. 280 and 281 have been prepared, and by the following appendix.

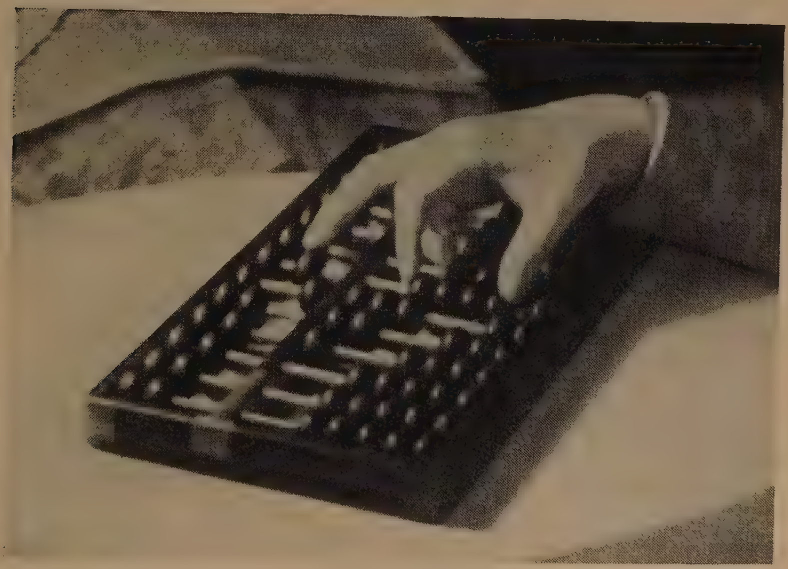
APPENDIX

The following description on the use of the abacus was kindly written for the Author by C. W. YU, who checked the trial-and-adjustment calculations included in the Paper. He demonstrated that the work could be carried out more quickly with an abacus than by the slide-rule or calculation machine.

The use of the Abacus

As seen from *Fig. 23*, the abacus consists of several columns of beads mounted on a rectangular frame. There are seven beads in each column, two above and five below the cross-bar, when the abacus is put in the right position. The principle underlying the

Fig. 23



Chinese Abacus

operation of the abacus is the representation of a number by means of beads. Any of the five beads below the cross bar will represent one unit, and any of the two above five units. To represent a number, any one of the columns may be chosen as the unit column so that the column first on the left will be the ten column, and the second the hundred column, and so on.

For example, to represent 173 on the abacus, push all the beads above the cross bar up to the upper edge and all the beads below the cross bar down to the lower edge, leaving the centre bar clear of beads. Then choose any column as the unit column and push three beads from below up to the cross bar; this will represent the 3 of the number. In the column next to the left push down one bead from above down to the cross bar, and two up from below; this will represent the 70 of the number. On the second column to the left of the unit column push up one bead; this will represent the 100 of the number. Thus 173 is fully represented on the abacus.

The Actual Operation

Addition.—For example, to add together 173 and 121, the procedure consists simply in first representing 173 on the abacus as explained above. Then, in the unit column, push one extra bead up for the 1 of 121. In the ten column, push two extra beads up for the 20 of 121, and finally in the hundred column, push one extra bead up from below for the 100 of 121. There are then two beads in the hundred column, one bead above the cross bar and four below in the ten column (altogether representing 9 units) and four beads below the cross bar in the unit column. This represents 294 and is the result of adding 173 to 121.

Multiplication.—To multiply 173 by 2, let 2 be the multiplier. Represent 173 on the abacus as before. Then multiply the digits of the various columns separately by the multiplier, starting first with the unit column. Now 2 by 3 is 6. After pushing down the three beads in the unit column, the product 6 is put down using the column next on the right of the original column as the unit column. Then multiply 7 by 2, which is 14. Remove the 7 from the original ten column and put back 1 and add four extra beads to the column first to the right (in other words, the product 14 is put down using the column next on the right of the multiplied column as the unit column). Next multiply 2 by 1 which is 2. Remove the 1 as before and add two beads to the column first to the right. This will leave 346 represented on the abacus, which is the product of 173 by 2.

With a little practice and skill both the operations can be performed very rapidly. The advantage of the abacus is that in a complicated calculation every stage is registered before the operator leaving no chance to the memory.

Paper No. 5893

"Reservoir Regulation Problems"

by

Robert Mathieson, B.Sc., Ph.D.

(Ordered by the Council to be published in abridged form.)

IN recent years the necessity of taking measures for flood prevention, together with the importance of efficiently utilizing all water resources, has led to the planning and construction of numerous storage projects throughout the world. Many of the problems associated with such projects lead to a consideration of the general reservoir equation:

Rate of storage = rate of inflow — rate of outflow

or
$$A\frac{dh}{dt} = Q - q \qquad (1)$$

where the surface area A and the outflow q may vary with the height h of the surface above some datum, while the inflow Q usually varies with time.

The Generalized Gould's Function [1] gives a convenient solution when the inflow is uniform and the outflow $q = Kh^n$.

Consideration is now given to some problems chosen to illustrate the development of analytical methods for use when equation (1) takes a more complicated form, for analytical solutions, when available, are frequently preferable to and less tedious than numerical or graphical [2, 3, 4] methods.

Problem Type 1 (Fig. 1)

When the inflow to the reservoir is zero, integration is possible with outflow through two pipes. (These may be replaced by undersluices, orifices, or siphon spillways.) With constant area:

$$t = \text{Const.} - A\int\frac{dh}{k_1\sqrt{h} + k_2\sqrt{h + h_1}} \qquad (2)$$

$$\therefore t = \text{Const.} + \frac{2k_1A}{b}\left[-\sqrt{h} + \sqrt{\frac{a}{b}}\tanh^{-1}\sqrt{\frac{b}{a}h}\right]$$
$$+ \frac{2k_2A}{b}\left[+\sqrt{h + h_1} - \sqrt{\frac{c}{b}}\tanh^{-1}\sqrt{\frac{b}{c}(h + h_1)}\right] \qquad (3)$$

---

[1] The references are given on p. 322.

where $a = k_1^2 h_1$; $b = k_1^2 - k_2^2 > 0$; and $c = k_2^2 h_1$

$$\text{or } t = \text{Const.} + \frac{2k_1 A}{b}\left[-\sqrt{h} + \sqrt{-\frac{a}{b}}\tan^{-1}\sqrt{-\frac{b}{a}h}\right]$$

$$+ \frac{2k_2 A}{b}\left[+\sqrt{h+h_1} - \sqrt{-\frac{c}{b}}\tan^{-1}\sqrt{-\frac{b}{c}(h+h_1)}\right] \quad . \quad (4)$$

when $b < 0$

and if $b = 0$

$$t = \text{Const.} + \frac{2A}{3k_1 h_1}\left[h^{\frac{3}{2}} - (h+h_1)^{\frac{3}{2}}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Integration is possible if A is a polynomial function of h, but the number of terms involved increases rapidly with the degree in h.

Fig. 1

Problem Type 2 (*Fig.* 2)

Consider a reservoir with constant area A, constant inflow Q, and outflow over a rectangular weir and through pipes for which the effective heads $(h + h_\beta)$ are assumed to be large compared with h but not large enough to assume their flows constant. Then :

$$A\frac{dh}{dt} = Q_o - Kh^{\frac{3}{2}} - \Sigma k_\beta \sqrt{h + h_\beta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Taking $\sum k_\beta \sqrt{h + h_\beta} \risingdotseq \sum \beta\sqrt{h_\beta} + h\sum \frac{k_\beta}{2\sqrt{h_\beta}} = a + bh$ (say) and putting $h = u^2$, then :

$$t = \text{Const.} - 2A\int\frac{u\,du}{Ku^3 + bu^2 + (a - Q_0)} \quad . \quad . \quad . \quad . \quad (7)$$

Case I.—Writing $p = -\frac{b^2}{q}$ and $r = -\left[\frac{K^2}{2}(a - Q_0) + \frac{b^2}{27}\right]$ then, if $p^3 + r^2 \leqslant 0$, one of the three roots of the denominator is :

$$u_1 = \frac{1}{K}\left\{\pm 2\sqrt{-p}\cos\left[\tfrac{1}{3}\cos^{-1}\frac{\pm r}{\sqrt{-p^3}}\right] - \frac{b}{3}\right\}$$

the lower sign being used when r is negative. Equation (7) now takes the form :

$$t = \text{Const.} - 2A\int\frac{u\,du}{K(u-u_1)(u^2+mu+n)}$$

When $4n > m^2$

$$t = \text{Const.} - \frac{2A}{K(n+u_1{}^2+nu_1)}\left\{u_1\log_e\frac{\sqrt{h}-u_1}{\sqrt{h+m\sqrt{h}+n}}\right.$$

$$\left.+\frac{u_1m+2n}{\sqrt{4n-m^2}}\tan^{-1}\frac{2\sqrt{h}+m}{\sqrt{4n-m^2}}\right\} \quad . \quad . \quad . \quad . \quad (8)$$

If $4n < m^2$ the $\tan^{-1}$ term becomes

$$\frac{u_1m+2n}{\sqrt{m^2-4n}}\tanh^{-1}\frac{2\sqrt{h}+m}{\sqrt{m^2-4n}}$$

and if $4n = m^2$ the same term becomes $-\dfrac{u_1m+2n}{2\sqrt{h}+m}$

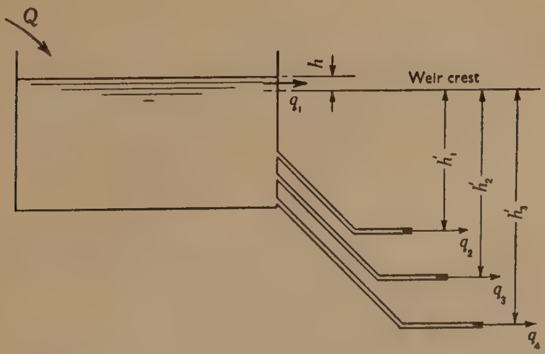
Case II.—If $p^3 + r^2 \geqslant 0$ the only real root is :

$$u_1 = \frac{1}{K}\left\{\pm 2\sqrt{-p}\cosh\left[\tfrac{1}{3}\cosh^{-1}\frac{\pm r}{\sqrt{-p^3}}\right] - \frac{b}{3}\right\}$$

whereafter the solution is the same as for Case I, resulting in equation (8).

As with Problem Type 1, integration is possible when A is a polynomial function of h.

Fig. 2



Problem Type 3 (Fig. 3)

A linearly varying inflow $Q = Q_0 \pm Q_1t$ with outflow through a number of pipes is now considered. By choosing a datum level below the surface so that the outflow can be written :

21

$$q \simeq \sum k_\beta \sqrt{h_\beta} + h \sum \frac{k_\beta}{2\sqrt{h_\beta}}$$

and assuming A to be constant:

$$A\frac{dh}{dt} = Q_0 \pm Q_1 t - \sum k_\beta \sqrt{h_\beta} - h \sum \frac{k_\beta}{2\sqrt{h_\beta}} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

This is a linear differential equation which gives:

$$h = \frac{a}{b} \pm \frac{Q_1}{b}\left(t - \frac{a}{b}\right) + Ce^{-\frac{b}{a}t} \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

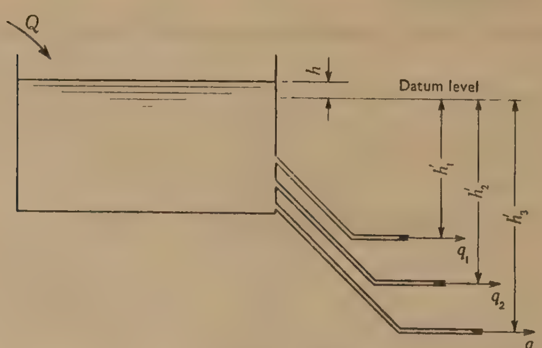
where $a = Q_0 - \sum k_\beta \sqrt{h_\beta}$; $b = \sum \frac{k_\beta}{2\sqrt{h_\beta}}$; and C is an arbitrary constant. Solutions can be obtained if A is a linear function of h or, alternatively, if Q is a polynomial of higher degree in t.

Problem Type 4

Several of the cases in reference 1 (p. 322) are considered with a linearly varying inflow. Taking the reservoir area A to be constant, the equation involved is:

$$A\frac{dh}{dt} + Kh^n = Q_0 \pm Q_1 t \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Fig. 3



Case I.—In this case $n = \frac{1}{2}$ which corresponds to discharge by siphon spillway, undersluice, or pipe. The variables are separated by putting $u = \sqrt{h}/v$ and $v = Q_0 \pm Q_1 t$.
Then, with $K^2 \pm 8AQ_1 < 0$:

$$\log v = \text{Const.} - \left\{ \tfrac{1}{2} \log_e(\pm 2AQ_1 u^2 + Ku - 1) - \frac{K}{\sqrt{-(K^2 \pm 8AQ_1)}} \tan^{-1} \frac{\pm 4AQ_1 u + K}{\sqrt{-(K^2 \pm 8AQ_1)}} \right\} \quad . \quad (12)$$

If $K^2 \pm 8AQ_1 > 0$ the $\tan^{-1}$ term becomes :

$$-\frac{K}{\sqrt{K^2 \pm 8AQ_1}}\tanh^{-1}\frac{\pm 4AQ_1u + K}{\sqrt{K^2 \pm 8AQ_1}}$$

and if $K^2 = 8AQ_1$ the same term becomes $+\dfrac{K}{4AQ_1u - K}$

It is convenient to leave the solution in this form and insert values of $v = Q_0 \pm Q_1t$ and $u = \sqrt{h}/(Q_0 \pm Q_1t)$ as required.

Case II.—In this case $n = 1$, which corresponds to discharge by lamina pipe-flow. The solution is :

$$h = \frac{Q_0}{K} \pm \frac{Q_1}{K}\left(t - \frac{A}{K}\right) + \text{Const.} \times e^{-\frac{K}{A}t} \quad . \quad . \quad . \quad . \quad (13)$$

Case III.—When the discharge is over a parabolic U-type notch, $n = 2$. Putting $h = \pm\frac{1}{u}\cdot\frac{du}{dx}$; $x = \frac{K}{Q_1A}(Q_0 \pm Q_1t)$; and $p = \frac{Q_1A}{K^2}$

then
$$\frac{d^2u}{dx^2} = pxu \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Solving this in terms of Modified Bessel Functions :

$$u = x^{\frac{1}{2}}[MI_{\frac{1}{3}}(\tfrac{2}{3}\sqrt{p}\,.\,x^{\frac{3}{2}}) + NI_{-\frac{1}{3}}(\tfrac{2}{3}\sqrt{p}\,.\,x^{\frac{3}{2}})],$$

M and N being constants ; so that :

$$h = \pm\frac{1}{u}\cdot\frac{du}{dx}$$
$$= \pm\left\{\frac{1}{2x} + \sqrt{p}.x^{\frac{1}{2}}\left[\frac{I_{\frac{1}{3}}'(\frac{2}{3}\sqrt{p}.x^{\frac{3}{2}}) + CI'_{-\frac{1}{3}}(\frac{2}{3}\sqrt{p}.x^{\frac{3}{2}})}{I_{\frac{1}{3}}(\frac{2}{3}\sqrt{p}.x^{\frac{3}{2}}) + CI_{-\frac{1}{3}}(\frac{2}{3}\sqrt{p}.x)^{\frac{3}{2}}}\right]\right\} \quad (15)$$

Values of $I_{\pm\frac{1}{3}}(z)$ have been tabulated,[5] whilst values of $I_{\pm}{}'_{\frac{1}{3}}(z)$ can be calculated by means of the recurrence formulae :

$$I_{\frac{1}{3}}'(z) = -\frac{1}{3z}I_{\frac{1}{3}}(z) + I_{-\frac{2}{3}}(z)$$

and
$$I_{-\frac{1}{3}}'(z) = -\frac{1}{3z}I_{-\frac{1}{3}}(z) + I_{\frac{2}{3}}(z)$$

However, an alternative solution is given in the Appendix, together with tabulated values sufficient for most problems of the type considered. This solution gives :

$$h = \pm\frac{\phi_1(Z) + \frac{C}{x}\phi_2(Z)}{x\phi_3(Z) + C\phi_4(Z)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

where C is a constant of integration. Values of $\phi(Z)$ for $Z = px^3$ are given in Table 1.

Fig. 4

FLOOD HYDROGRAPH

Case IV.—For discharge over a rectangular weir $n = \frac{3}{2}$ and the case with linearly varying inflow is of some importance in view of the suggestion by the Committee on Floods in Relation to Reservoir Practice [6] that a hydrograph of the type shown in *Fig. 4* be used in calculations for flood control.

The equation falls into a class for which there is no general method of solution and the approximate method of Richards,[7] which gives the maximum value of head with a trapezoidal hydrograph, is the only one known. The following approximate solution appears to be accurate enough for most purposes.

The curve of $y = h^{\frac{3}{2}}$ can be closely represented over a limited range by the curve $y_1 = ah + bh^2$, where a and b are constants to be chosen. The fractional error in representation is given by $E = ah^{-\frac{1}{2}} + bh^{-\frac{1}{2}} - 1$ and if it is assumed that an accuracy of $\pm$ 100ϵ per cent is required over a range for which the upper limit is H, then with $E = +\epsilon$ at $h = H$:

$$(1 + \epsilon)H^{\frac{1}{2}} = a + bH \quad . \quad . \quad . \quad . \quad (17)$$

and the rate of change of E with h will be zero when $\frac{d(E)}{dh} = 0$ or $h = \frac{a}{b}$; and with $E = -\epsilon$ at $h = \frac{a}{b}$ then the equation obtained is:

$$(1 + \epsilon) = 2\sqrt{ab} \quad . \quad . \quad . \quad . \quad . \quad (18)$$

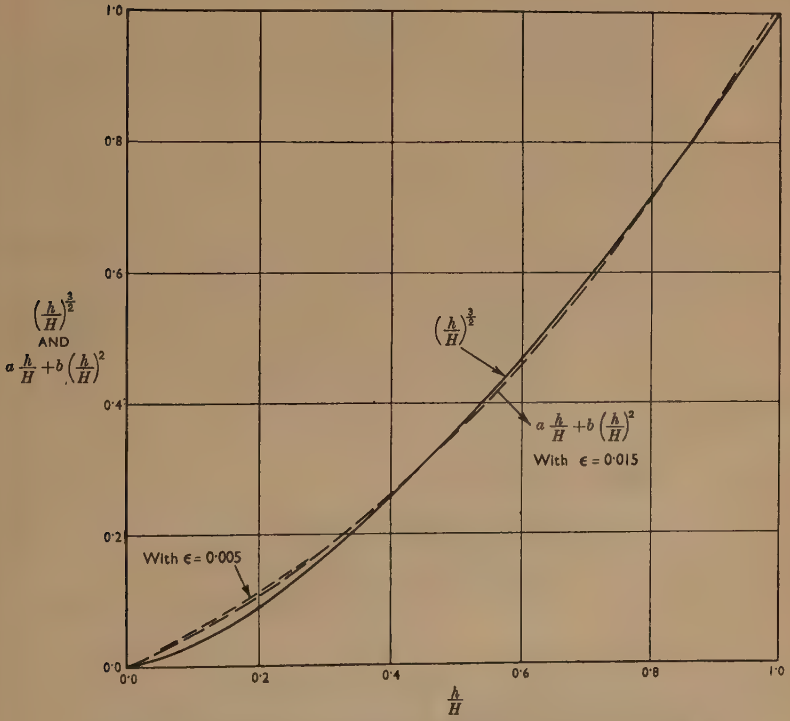
Solving (17) and (18) for a and b:

$$a = \frac{H^{\frac{1}{2}}}{2}(1 - \epsilon^{\frac{1}{2}})^2 \text{ and } b = \frac{(1 + \epsilon^{\frac{1}{2}})^2}{2H^{\frac{1}{2}}} \quad . \quad . \quad . \quad . \quad (19)$$

and the lower limit of the range to give the same accuracy is found to be $\frac{1}{H}\left(\frac{a}{b}\right)^2$

Table 2 indicates the accuracy and ranges of representation possible and Table 3 gives values of a and b in terms of selected H-values. The measure of agreement shown by the curves in *Fig. 5* suggests that if H is not too high the substitution could be used for the full range 0 to H. For more accurate work a given range can be covered in two or more stages.

Fig. 5



Dimensionless Curves showing the Approximation to $y = h^{\frac{3}{2}}$

Substituting $ah + bh^2$ for $h^{\frac{3}{2}}$ in equation (11) and putting

$$h = \pm\frac{1}{u}\cdot\frac{du}{dx} - \frac{a}{2b}; \quad x = \frac{bK}{A}\left(\frac{Q_0}{Q_1} + \frac{a^2K}{4bQ_1} \pm t\right); \text{ and } p = \frac{Q_1 A}{b^2 K^2}$$

then equation (11) becomes

$$\frac{d^2u}{dx^2} = pxu \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

This is the same equation as (14) and therefore alternative solutions are obtained as :

$$h = \pm\left\{\frac{1}{2x} + \sqrt{p}x^{\frac{1}{2}}\left[\frac{I_{\frac{1}{3}}'(\frac{2}{3}\sqrt{p}x^{\frac{3}{2}}) + CI_{-\frac{1}{3}}'(\frac{2}{3}\sqrt{p}x^{\frac{3}{2}})}{I_{\frac{1}{3}}(\frac{2}{3}\sqrt{p}x^{\frac{3}{2}}) + CI_{-\frac{1}{3}}(\frac{2}{3}\sqrt{p}x^{\frac{3}{2}})}\right]\right\} - \frac{a}{2b} \quad . \quad . \quad . \quad (21)$$

$$\text{or } h = \pm\left\{\frac{\phi_1(z) + \frac{C}{x}\phi_2(z)}{x\phi_3(z) + C\phi_4(z)}\right\} - \frac{a}{2b} \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

where C is a constant of integration and the lower alternative sign applies when Q_1 is negative.

The approximate substitution enables equation (11) to be solved when the inflow Q takes more complicated forms. If the inflow is expressible in the form of a power series in t so that :

$$A\frac{dh}{dt} + Kah + Kbh^2 = Q_0 + Q_1t + Q_2t^2 + \quad . \quad . \quad (23)$$

then the substitution $h = \frac{A}{Kb}\cdot\frac{1}{u}\cdot\frac{du}{dt}$ gives a linear second-order equation which is solvable by well known methods, the solution being of the type $u = C_1u_1 + C_2u_2$. The solution of equation (19) will therefore be of the form :

$$h = \frac{A}{Kb}\left[\frac{u_1' + Cu_2'}{u_1 + Cu_2}\right]$$

where $C = \frac{C_2}{C_1}$ is a constant.

If the inflow varies sinusoidally, so that :

$$A\frac{dh}{dt} + Kah + Kbh^2 = Q_0 + Q_1 \sin \omega t \quad . \quad (24)$$

then, on putting $h = \frac{A}{Kb}\cdot\frac{1}{u}\cdot\frac{du}{dt}$ and $t = \frac{2}{\omega}\left(z + \frac{\pi}{4}\right)$, equation (24) reduces to Mathieu's equation :

$$\frac{d^2u}{dz^2} + (a' - 2q' \cos 2z)u = 0 \quad . \quad . \quad (25)$$

where $a' = -\frac{4Kb}{\omega^2A^2}\left(\frac{Ka^2}{4b} + Q_0\right)$ and $q' = \frac{2KbQ_1}{\omega^2A^2}$

Reference 8 gives a chart of the (a', q') plane divided into regions and the position of the point (a', q') on the chart determines the nature of the solution. Here a' will be negative and q' positive and, for instance, if the point (a', q') is assumed to fall within an unstable zone then the solution of equation (25) will take the form :

$$u = C_1 e^{\mu z} \sum_{r=0}^{\infty} \rho_m \cos (mz + \phi_m) + C_2 e^{-\mu z} \sum_{r=0}^{\infty} \rho_m \cos (mz - \phi_m)$$

$$\text{and } h = \frac{A}{Kb} \left\{ \frac{e^{\mu z} \sum_{r=0}^{\infty} \rho_m [\mu \cos (mz + \phi_m) - m \sin (mz + \phi_m)] + Ce^{-\mu z} \sum_{r=0}^{\infty} \rho_m [-\mu \cos (mz - \phi_m) - m \sin (mz - \phi_m)]}{e^{\mu z} \sum_{r=0}^{\infty} \rho_m \cos (mz + \phi_m) + Ce^{-\mu z} \sum_{r=0}^{\infty} \rho_m \cos (mz - \phi_m)} \right\} \quad . \quad . \quad . \quad (26)$$

where ρ_m, ϕ_m, and μ have values determined by (a', q') and $m = 2r$ or $2r + 1$ depending on the particular unstable zone concerned.

If now (a', q') lies within a stable zone, the corresponding solution would be :

$$h = \frac{A}{Kb} \left\{ \frac{-\sum_{r=-\infty}^{+\infty} C_m (m + \beta) \sin (m + \beta) z + C \sum_{r=-\infty}^{+\infty} C_m (m + \beta) \cos (m + \beta) z}{\sum_{r=-\infty}^{+\infty} C_m \cos (m + \beta) z + C \sum_{r=-\infty}^{+} C_m \sin (m + \beta) z} \right\} \quad . \quad . \quad . \quad (27)$$

where C_m and β have values determined by (a', q') and $m = 2r$ or $2r + 1$ depending on which stable zone is concerned.

Case V.—For discharge over a Vee notch $\left(n = \frac{5}{2}\right)$, equation (11) is not integrable, but approximate substitutions of reasonable

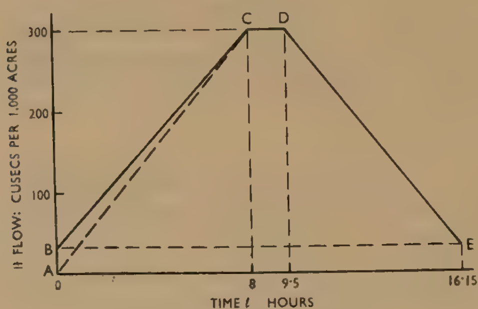
accuracy can be obtained to cover limited ranges. For instance, taking $h^{\frac{5}{2}} \simeq d + ah + bh^2$ then, with $d = 0{\cdot}08317H^{\frac{5}{2}}$, $a = -0{\cdot}5520H^{\frac{3}{2}}$, and $b = 1{\cdot}46838H^{\frac{1}{2}}$, the representation (which is suitable for use with an initial discharge) would cover the range $0{\cdot}6H$ to H within ± 1 per cent.

Example

To illustrate the use of the $\phi(z)$ method (equation (22)) the following data are assumed:

Flood hydrograph: as in *Fig. 6*.
Reservoir area: $2{\cdot}178 \times 10^6$ square feet per 1,000 acres of catchment.
Spillway discharge: $86{\cdot}58h^{\frac{3}{2}}$ cusecs.
Initial flood: 30 cusecs (0·4933 foot head on spillway).

Fig. 6



FLOOD HYDROGRAPH

Considering the line BC in *Fig. 6*, and assuming that the head will not exceed 1·75 foot after the first 8 hours, this period can be covered in one step with $\epsilon = 0{\cdot}025$, $H = 1{\cdot}75$ foot, $a = 0{\cdot}46882$ and $b = 0{\cdot}50692$. Working in cubic feet and hour units, $Q_0 = 1{\cdot}08 \times 10^5$ cubic feet per hour, $Q_1 = 1{\cdot}215 \times 10^6$ cubic feet per hour per hour, and $K = 3{\cdot}1169 \times 10^5$. Then:

$$x = \frac{bK}{A}\left(\frac{a^2K}{4bQ_1} + \frac{Q_0}{Q_1} + t\right) = 0{\cdot}084658 + 0{\cdot}072546t$$

$$p = \frac{Q_1 A}{b^2K^2} = 10{\cdot}6002.$$

When $t = 0$, $h = 0{\cdot}4933$, $x = 0{\cdot}08466$, $z = px^3 = 0{\cdot}006432$, and, using the values of $\phi(z)$ from Table 1 in equation (22):

$$0{\cdot}4933 = + \frac{1{\cdot}00214 + C \times \dfrac{0{\cdot}003216}{0{\cdot}08466}}{0{\cdot}08466 \times 1{\cdot}00054 + C \times 1{\cdot}00107} - 0{\cdot}46242$$

which gives $C = 1{\cdot}0027$.

When $t = 8$ hours, $x = 0{\cdot}66503$, $z = 3{\cdot}11773$ and :

$$h = \frac{2{\cdot}181102 + 1{\cdot}0027 \times \dfrac{1{\cdot}9049}{0{\cdot}66503}}{0{\cdot}66503 \times 1{\cdot}27977 + 1{\cdot}0027 \times 1{\cdot}576015} - 0{\cdot}46242 = 1{\cdot}61574.$$

Values of h at $t = 2$, 4, and 6 hours have been calculated for comparison with step-by-step results (using the average inflow and outflow during 15-minute intervals) in Table 4.

To cover the period C to D ($8 - 9{\cdot}5$ hours), since Q ($= 300$ cusecs) is constant, Gould's Function applies and, using the $\phi(r)$ values tabulated in reference 1, with $H_1 = \left(\dfrac{300}{86{\cdot}58}\right)^{\frac{2}{3}} = 2{\cdot}2898$, $r_1 = \dfrac{h}{H_1} = 1{\cdot}61574$ and $\phi(r_1) = 0{\cdot}9866$; then :

$$t_2 - t_1 = \frac{A}{KH_1^{\frac{1}{2}}}[\phi(r_2) - \phi(r_1)]$$

gives $\phi(r_2) = 0{\cdot}9866 + 0{\cdot}32483 = 1{\cdot}31143$ and $r_2 = 0{\cdot}8135$. Therefore, at D, 9·5 hours from the start, $h = 1{\cdot}86274$ foot.

For the period D to E, new values have to be selected for a and b. Taking $H = 2$ feet, to allow for the head still rising, with $\epsilon = 0{\cdot}025$, then $a = 0{\cdot}501178$ and $b = 0{\cdot}474197$; $Q_0 = 10{\cdot}8 \times 10^5$ cubic feet per hour and $Q_1 = 1{\cdot}38857 \times 10^5$ cubic feet per hour per hour and is negative—this is taken care of in the equations provided that the lower alternative sign is used. Taking a new time zero at D for convenience :

$$x = \frac{bK}{A}\left(\frac{a^2K}{4bQ_1} + \frac{Q_0}{Q_1} - t\right) = 0{\cdot}54798 - 0{\cdot}07861t$$

$$p = \frac{Q_1A}{b^2K^2} = 13{\cdot}8442$$

When $t = 0$, $h = 1{\cdot}86274$ foot, $x = 0{\cdot}54798$, and $z = px^3 = 2{\cdot}27804$.

$$\therefore h = 1{\cdot}86274 = -\frac{1{\cdot}83408 + C\dfrac{1{\cdot}32040}{0{\cdot}54798}}{0{\cdot}54798 \times 1{\cdot}200398 + C \times 1{\cdot}40943} - 0{\cdot}52845$$

giving $C = -0{\cdot}58947$.

When $t = 7$ hours, $x = 0{\cdot}07295$ and $z = 0{\cdot}005375$

$$\therefore h = -\frac{1{\cdot}00179 - 0{\cdot}58947 \times \dfrac{0{\cdot}002688}{0{\cdot}07295}}{0{\cdot}07295 \times 1{\cdot}00045 - 0{\cdot}58947 \times 1{\cdot}0009} - 0{\cdot}52845 = 1{\cdot}36716$$

This calculation has been repeated with $\epsilon = 0{\cdot}0075$, requiring two steps from B to C, with the results shown in Table 4.

As a further example the same reservoir data have been considered

together with the hydrograph shown in broken line from A to C in *Fig. 6*, and the results from the step-by-step and $\phi(z)$ methods for the first 8 hours compared in Table 5. Again the agreement appears to be satisfactory.

References

1. Robert Mathieson, "The Generalized Gould's Function." Proc. Instn Civ. Engrs, Part III, vol. 2, p. 142 (April, 1953).
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4. A. Lienard, "Étude des Oscillations entretenues" ("The Study of Sustained Oscillations"). *Rev. Gen. Élect.*, vol. 23 (1928), p. 901.
5. National Bureau of Standards, Tables of Bessel Functions of Fractional Order, Vols I and II. Columbia Univ. Press, 1949.
6. Committee on Floods in Relation to Reservoir Practice, Interim Report. Instn Civ. Engrs, 1933.
7. B. D. Richards, "Flood Hydrographs." J. Instn Civ. Engrs, vol. 5, p. 405 (Mar. 1937).
8. N. W. McLachlan, "Theory and Application of Mathieu Functions." Oxford Univ. Press, 1947.

The Paper is accompanied by six sheets of diagrams, from which the Figures in the text have been prepared, and by the following Appendix.

APPENDIX

CALCULATION OF THE z-FUNCTIONS

The solution in series of the second-order linear differential equation (14) is readily obtained using the well-known method (Frobenius) of assuming a solution

$$u = x^{\theta}(a_0 + a_1 x + a_2 x^2 + \ldots)$$

where a_0, a_1, a_2, etc. are constants and $a_0 \neq 0$. Differentiating twice and substituting for u and $\frac{d^2u}{dx^2}$ in equation (14)

$$\{a_0\theta(\theta-1)x^{-2} + a_1\theta(\theta+1)x^{-1} + a_2(\theta+1)(\theta+2) + a_3(\theta+2)(\theta+3)x + \ldots\}$$
$$= p\{a_0 x + a_1 x^2 + \ldots\}$$

Considering the coefficient of x^{-2}, if $a_0 \neq 0$ then $\theta(\theta-1) = 0$ and either $\theta = 0$ or $\theta = 1$. The values of θ differ by an integer, and here the complete solution of (16) is obtained by taking $\theta = 0$. Equating coefficients of x it is found that a_1 is indeterminate, $a_2 = 0$, $a_3 = \frac{pa_0}{2\,.\,3}$, $a_4 = \frac{pa_1}{3\,.\,4}$ and so on. The final solution is

$$u = a_0\left[1 + \frac{px^3}{2\,.\,3} + \frac{p^2x^6}{2\,.\,3\,.\,5\,.\,6} + \ldots\right] + a_1 x\left[1 + \frac{px^3}{3\,.\,4} + \frac{p^2x^6}{3\,.\,4\,.\,6\,.\,7} + \ldots\right]$$
$$= a_0\phi_4(z) + a_1 x\phi_3(z) \text{ with } z = px^3.$$

The series are convergent for all values of z except ∞.

$$\text{Also } \frac{du}{dx} = \frac{a_0}{x}\left[\frac{px^3}{2} + \frac{p^2x^6}{2\,.\,3\,.\,5} + \ldots\right] + a_1\left[1 + \frac{px^3}{3} + \frac{p^2x^6}{3\,.\,4\,.\,6} + \ldots\right]$$
$$= \frac{a_0}{x}\phi_2(z) + a_1\phi_1(z)$$

$$\text{Therefore } h = \frac{\phi_1(z) + \frac{C}{x}\phi_2(z)}{x\phi_3(z) + C\phi_4(z)} \quad \text{with } C = \frac{a_0}{a_1} \text{ a constant.}$$

Thus the four functions for which values are given in Table 1 were computed from

$$\phi_1(z) = 1 + \frac{z}{3} + \frac{z^2}{3\,.\,4\,.\,6} + \frac{z^3}{3\,.\,4\,.\,6\,.\,7\,.\,9} + \ldots$$

$$\phi_2(z) = \frac{z}{2} + \frac{z^2}{2\,.\,3\,.\,5} + \frac{z^3}{2\,.\,3\,.\,5\,.\,6\,.\,8} + \ldots$$

$$\phi_3(z) = 1 + \frac{z}{3\,.\,4} + \frac{z^2}{3\,.\,4\,.\,6\,.\,7} + \frac{z^3}{3\,.\,4\,.\,6\,.\,7\,.\,9\,.\,10} + \ldots$$

$$\phi_4(z) = 1 + \frac{z}{2\,.\,3} + \frac{z^2}{2\,.\,3\,.\,5\,.\,6} + \frac{z^3}{2\,.\,3\,.\,5\,.\,6\,.\,8\,.\,9} + \ldots$$

TABLE 1.—VALUES OF $\phi(z)$

$z = px^3$	$\phi_1(z)$	$\phi_2(z)$	$\phi_3(z)$	$\phi_4(z)$
6·0	3·55008	4.35942	1·57638	2·21745
5·9	3·49771	4·26176	1·56544	2·19330
5·8	3·44571	4·16505	1·55455	2·16929
5·7	3·39407	4·06927	1·54370	2·14542
5·6	3·34280	3·97443	1·53290	2·12170
5·5	3·29188	3·88051	1·52215	2·09811
5·4	3·24132	3·78751	1·51145	2·07466
5·3	3·19112	3·69543	1·50080	2·05135
5·2	3·14127	3·60427	1·49019	2·02818
5·1	3·09177	3·51401	1·47963	2·00514
5·0	3·04262	3·42464	1·46912	1·98224
4·9	2·99383	3·33618	1·45865	1·95948
4·8	2·94538	3·24862	1·44823	1·93685
4·7	2·89728	3·16195	1·43786	1·91436
4·6	2·84952	3·07616	1·42754	1·89200
4·5	2·80211	2·99124	1·41725	1·86978
4·4	2·75504	2·90719	1·40701	1·84769
4·3	2·70831	2·82401	1·39682	1·82573
4·2	2·66192	2·74169	1·38668	1·80390
4·1	2·61586	2·66023	1·37658	1·78221
4·0	2·57014	2·57962	1·36653	1·76065
3·9	2·52476	2·49986	1·35652	1·73922
3·8	2·47971	2·42094	1·34656	1·71791
3·7	2·43499	2·34285	1·33664	1·69674
3·6	2·39060	2·26560	1·32677	1·67570
3·5	2·34654	2·18918	1·31694	1·65479
3·4	2·30280	2·11358	1·30715	1·63400
3·3	2·25938	2·03880	1·29741	1·61334
3·2	2·21630	1·96484	1·28772	1·59281
3·1	2·17354	1·89168	1·27807	1·57241
3·0	2·13110	1·81933	1·26846	1·55213
2·9	2·08898	1·74778	1·25890	1·53198
2·8	2·04718	1·67702	1·24938	1·51195
2·7	2·00569	1·60705	1·23991	1·49205
2·6	1·96452	1·53786	1·23047	1·47227
2·5	1·92366	1·46946	1·22108	1·45262
2·4	1·88311	1·40184	1·21174	1·43309
2·3	1·84287	1·33498	1·20243	1·41368
2·2	1·80295	1·26889	1·19317	1·39439
2·1	1·76333	1·20357	1·18396	1·37523
2·0	1·72402	1·13900	1·17478	1·35618
1·9	1·68501	1·07519	1·16565	1·33726
1·8	1·64631	1·01213	1·15655	1·31846
1·7	1·60790	0·94981	1·14750	1·29977
1·6	1·56980	0·88822	1·13850	1·28121
1·5	1·53200	0·82738	1·12954	1·26276
1·4	1·49450	0·76727	1·12062	1·24444
1·3	1·45730	0·70788	1·11174	1·22623
1·2	1·42039	0·64922	1·10290	1·20813
1·1	1·38377	0·59127	1·09410	1·19016
1·0	1·34744	0·53403	1·08534	1·17230
0·9	1·31141	0·47751	1·07662	1·15456
0·8	1·27567	0·42169	1·06795	1·13693
0·7	1·24022	0·36657	1·05931	1·11941
0·6	1·20505	0·31215	1·05072	1·10201
0·5	1·17017	0·25842	1·04217	1·08473
0·4	1·13557	0·20538	1·03365	1·06756
0·3	1·10126	0·15302	1·02518	1·05050
0·2	1·06722	0·10134	1·01675	1·03356
0·1	1·03347	0·05033	1·00835	1·01672
0	1·0	0	1·0	1·0

TABLE 2

Accuracy (per cent)	Range	a	b
± 0·5	0·5520 H to H	0·428338 $H^{\frac{1}{2}}$	0·576662 $H^{-\frac{1}{2}}$
± 0·75	0·4993 H to H	0·41715 $H^{\frac{1}{2}}$	0·59035 $H^{-\frac{1}{2}}$
± 1·0	0·4481 H to H	0·405 $H^{\frac{1}{2}}$	0·605 $H^{-\frac{1}{2}}$
± 1·5	0·3735 H to H	0·385026 $H^{\frac{1}{2}}$	0·629974 $H^{-\frac{1}{2}}$
± 2·0	0·3201 H to H	0·36857 $H^{\frac{1}{2}}$	0·65143 $H^{-\frac{1}{2}}$

TABLE 3.—VALUES OF a AND b

Maximum head, H: feet		Accuracy: per cent			
		± 0·5	± 1	± 1·5	± 2
0·5	a	0·30288	0·28638	0·27225	0·26062
	b	0·81551	0·85560	0·89091	0·92126
0·75	a	0·37096	0·35074	0·33344	0·31919
	b	0·66587	0·69859	0·72742	0·75220
1·0	a	0·42834	0·40500	0·38502	0·36875
	b	0·57666	0·60500	0·62997	0·65143
1·25	a	0·49006	0·45279	0·43045	0·41206
	b	0·51580	0·54114	0·56348	0·58267
1·50	a	0·52467	0·49608	0·47161	0·45146
	b	0·47078	0·49392	0·51430	0·53182
1·75	a	0·56665	0·53577	0·50934	0·48758
	b	0·43591	0·45733	0·47620	0·49243
2·0	a	0·60576	0·57275	0·54450	0·52123
	b	0·40776	0·42780	0·44546	0·46063
2·5	a	0·67725	0·64035	0·60876	0·58275
	b	0·36472	0·38264	0·39844	0·41201
3·0	a	0·74193	0·70150	0·66689	0·63840
	b	0·33293	0·34929	0·36370	0·37609
3·5	a	0·80134	0·75767	0·72030	0·68952
	b	0·30824	0·32339	0·33674	0·34821
4·0	a	0·85668	0·81000	0·77004	0·73714
	b	0·28833	0·30250	0·31499	0·32572

Table 4.—Results of calculations

Time: hours	Water level, h: feet		
	Step-by-step method	$\phi(z)$ method: $\epsilon = 0{\cdot}025$	$\phi(z)$ method: $\epsilon = 0{\cdot}0075$
0	0·4933	0·4933	0·4933
2	0·5943	0·5940	0·5872
4	0·8551	0·8424	0·8561
6	1·2132	1·2092	1·2254
8	1·6122	1·6019	1·6157
9·5	1·8613	1·8536	1·8627
11	1·9594	1·9534	1·9616
11·5	1·9555	1·9491	1·9449
13·5	1·8153	1·8099	1·8066
15·5	1·5411	1·5405	1·5427
16·5	1·3707	1·3778	1·3672

Table 5.—Results with no initial discharge

Time: hours	head, h: feet	
	Step-by-step method	$\phi(z)$ method $\epsilon = 0{\cdot}015$
0	0	0
2	0·1201	0·1198
4	0·4520	0·4441
6	0·9099	0·9058
8	1·4163	1·4149

CORRESPONDENCE

on Papers published in Proceedings, Part III, April 1953

Structural Paper No. 33

" The Design and Construction of the British European Airways Hangars at London Airport, with Particular Reference to Prestressed Concrete " †

by

Dudley Holt New, B.Sc.(Eng.), M.I.C.E.

Correspondence

Mr E. Wingrove Keer enquired as to the nature of the grout used in the cable ducts of the post-tensioned beams. Was it of the colloidal kind or of plain mix ? What pressure had been needed to force it through the ducts of the 150-foot-span beams ?

Regarding the failure in one of the secondary beams, he would be interested to know if the mortar packing between the precast units had been kept saturated with water after the initial set of the mortar had taken place. Some such precaution would appear to be necessary to ensure complete hydration during hardening. It seemed possible that the failure might have occurred through an uneven distribution, over the sectional area, of the hardening of the mortar (by reason, perhaps, of insufficient water). The effect of that, after prestressing had been carried out, would be to impose on the plain end of the adjacent precast unit a number of point loads with resultant compressive overloading. That could cause crushing of the concrete. If any apprehension was felt on that point it would be interesting to know if the Author thought the difficulty could be overcome by arranging a thickening of the web and flange of the units at the plain ends for a short distance—say, from 4 inches to 6 inches for the last 9 inches or so? Such an arrangement would materially lower the compressive stress to be borne by the mortar joint at a practically negligible extra cost.

Could the Author explain the use of the steel lattice beam used in lifting the secondary beams ? *Fig 12* indicated that the assembly was slung from the ends only, and had no intermediate slings to the concrete beam. Why could not the latter have been lifted direct by the slings ? Was the purpose

† Proc. Instn Civ. Engrs, Part III, vol. 2, p. 15 (April 1953).

of the steel lattice to take the horizontal thrust component from the slings ?

Was there only one line of stiffening units across the secondary beams ? From a study of *Fig. 13* there appeared to be several such lines.

What was the diameter of the conduit enclosing the cable on the 150-foot and 110-foot-span beams ? That was particularly interesting in view of the very thin sections in which they were contained. Had any investigations been carried out to discover whether there might have been voids on the underside of the conduits ? Mr Keer had found that it was always difficult to secure the same standard of compaction immediately under a horizontal bar or tube.

From the Author's description it was not quite clear what form the wall shuttering had taken. Had the shuttering units spanned between the wing walls of the columns, and had horizontal stiffening members been bolted to those wing walls ? Such an arrangement would presumably have eliminated through-bolts or clamps.

What arrangements had been made to ensure that the cast-in-situ concrete walls and upper flange of the main beams acted monolithically ? Had the adhesion of the cast-in-situ concrete to the precast diaphragms been found to be sufficient to overcome the eventual tendency of the one to slip over the other ? Alternatively, had it been found necessary to incorporate bars or steps to counteract that tendency at the junctions ?

Could the Author say if any difficulty had been experienced in the selection of a suitable type of concrete-mixer for dealing with the stiff mixes necessary to produce the very-high-grade concrete called for in the main-beam in-situ work ? Mr Keer had found that the normal type of mixer could not be efficaciously used for such concrete, owing to the difficulty of ejecting the concrete from the drum. Had the Author encountered the same trouble and, if so, how had it been overcome ?

Mr J. S. Henzell of Melbourne, Australia, observed that the brilliant project described in the Paper had proved that prestressed concrete could replace steelwork for industrial building structures of up to 200 feet span. Maintenance would be negligible, since there was no steelwork tracery to be painted and protected above busy workshop floors during the years to come. The Author's prestressed concrete design required even less structural depth than would have been necessary for a competitive steelwork roof system. The only standpoint from which steel would still appear favourable was that of cost.

The Author had stated that, for that contract, which had been competitive, prestressed concrete had been favoured in a period of acute steel shortage. Did, therefore, prestressed concrete compete with steelwork in cost or time, separately or together ? If steelwork had been available, at normal price and normal delivery, would the sub-contract have gone to a steelwork fabricator ? The answers to those questions would be clarified if the Author would give brief quantities of the gross floor areas of the

structure, the steel weights thereof (for both hot-rolled and cold-drawn steel), the averaged volumes of concrete, and a clearer specification of the work included in the contract price for ground floor slabs (if any), framing, crane girders, cladding, glazing, sliding doors, paint or finish, and drainage. From those brief quantities a direct comparative cost per square foot could be derived for the two alternative construction methods.

It was noted that the roof framing for the 110-foot spans along the 900-foot length of the main shops required only 2 lb. of steel per square foot, excluding perimetral supports. That low figure was attractive—in fact it was incredible to those who still regarded prestressed concrete as just another variation of reinforced concrete—but it should be compared with steelwork costs as they were in practice. If steelwork had been specified with equivalent glazing, and contracted on a cost-plus basis, then the weight might have been 8 or even 10 lb. per square foot, if cross-valleyed to a limiting overall depth approaching that of the Author's roof. If not so limited in depth, and if engineered merely as low-cost coverage for asbestos-cement sheeting, using welded bar joists (American style, but to British code loadings), then the steel weight on the 110-foot spans could have been as low as 3·75 lb. per square foot. In either case, the hangars would have been in every way inferior to the Author's prestressed-concrete design.

Could the Author state the comparative first costs for steelwork and prestressed concrete for the project, assuming normal availability of materials and a design standard of overall height of structure acceptable to the owners ?

The Author, in reply to Mr Wingrove Keer, stated that a 1 : $1\frac{3}{4}$ cement/sand grout of plain mix had been used for the first beams, but owing to injection difficulties, that had been eventually discarded in favour of a neat cement grout, also of plain mix. Grout had been applied under a pressure of 80 lb. per square inch.

Steps had been taken to ensure adequate curing of the mortar in the dry-packed joints of the secondary beams. The Author's theory of the failure appeared to coincide closely with Mr Keer's. Whilst he agreed that a thickening of the web units would decrease the stress, he did not consider that that would have been either necessary or economic.

The purpose of the steel lattice beam was to take out the horizontal thrust set up by the slings and thus relieve the stress in the table of the concrete " Tee."

There was only one line of stiffening units across the secondary beams. The other units shown in Fig. 13 were to carry the aluminium roofing and roof lights, and whilst they would undoubtedly have a stiffening effect on the secondary beams, they were not designed with that in view.

The outside diameter of the conduits enclosing the cables was $1\frac{1}{4}$ inch. Some cutting away had been carried out on one beam to expose the cables, but there had been no evidence to show that the standard of compaction under them was unsatisfactory.

Through bolts had been used for positioning the shuttering units between the wing walls of the columns.

Normal jointing precautions had been taken between the cast-in-situ walls and the upper flange of the main beams, and, as in the case of composite construction, that was known to result in a structure that acted monolithically.

Following the original work of Freyssinet, it was now normal practice to form joints between precast elements with perfectly plain faces when those elements were prestressed together. There was no tendency to move at the junctions.

A normal 10/7 closed-drum mixer had been used for the highest grade of concrete. No difficulties had been experienced in ejecting the concrete from the drum.

Turning to Mr Henzell's comments, the Author observed that although tenders for schemes in reinforced concrete and steelwork had been submitted to the Ministry of Civil Aviation, prices had not been divulged, and the Author could only assume that the scheme submitted by his firm had been lower than others.

The answers to the remainder of the questions would go far beyond the scope envisaged by the Author, and although the information could be obtained it would involve a very considerable amount of work, and could in itself form the subject of a further Paper.

Public Health Paper No. 5

"The Storage, Collection, and Disposal of Domestic Refuse" †

by

Jesse Cooper Dawes, C.B.E., M.I.Mech.E.

Correspondence

Mr S. S. Morris, of Cape Town, stated that conditions in South Africa differed very materially from those obtaining in England and it was interesting to see how the problem appeared in the light of conditions prevailing in South Africa.

In England and Wales, 10 million tons of refuse was collected and disposed of annually at a cost of £16 million, or 32*s*. per ton. In Cape Town, approximately 125,000 tons was collected and disposed of for £200,000, the cost again being 32*s* per ton.

† Proc. Instn Civ. Engrs, Part III, vol. 2, p. 98 (April 1953).

The Author had pointed out that the disparity between the efficacy of the collection and disposal services was wasteful, and had suggested that a 70 per cent to 30 per cent division of expenditure was normal. The division in Cape Town would be approximately 75 per cent to 25 per cent.

In Cape Town, the receptacles used for the storage of refuse at self-contained dwelling houses conformed to the requirements stated in the Paper. No refuse bins for residential premises were stored indoors. A daily collection was made in business areas where that type of storage existed.

The scheme for bins being provided by the local authority seemed an excellent one ; it certainly obviated many difficulties at present encountered with insanitary and unsuitable containers. Mr Morris doubted, however, whether a cost of 5*s* annually, when small bins cost £1 10*s* each, was economical. It would be interesting to know on what price per bin that charge of 5*s* was assessed.

In Cape Town, large rectangular containers which were hoisted directly on to collection vehicles were employed at certain flats, hospitals, and depositions from street-sweeping orderly carts in the central city area. The vehicle employed held six such bins, each of 34 cubic feet capacity. The cost of this type of removal was high because the capacity of the vehicle was only 7½ cubic yards, and frequent trips to the disposal site had to be made. It had been found more economical for flats to utilize batteries of bins of 3 cubic feet capacity, which were rapidly emptied into the 14-cubic-yard-capacity collection vehicles.

According to the nature of the district, refuse was collected either daily or twice weekly, clearance being carried out at fixed times, but in areas where a daily collection was made, late afternoon collections were usually 1–1½ hour later than those elsewhere. On Saturdays, collections were earlier. Refuse was not weighed, but an accurate tally of the volume was kept, from which the weight could be estimated with fair accuracy.

All vehicles used in the city were completely enclosed and conformed to all other requirements laid down in the Paper under the heading " Refuse Collection Vehicles."

Refuse was disposed of in Cape Town by a single handling. Double handling had been discontinued in 1952 as being uneconomical. All precautions necessary to control insect and vermin infestations were taken by both the Cleansing Branch of the City Engineer's Department and the Health Department.

The city was expanding at such a rapid rate that it had not been possible to connect many new buildings in outlying areas to the sewers. It was estimated that about 13,000 properties had stercus removals. Over 13,000 tons of stercus were removed annually in 650,000 pails at a cost of £2 10*s* per ton for removal and disposal. Disposal was by two methods : by burying or by discharge at a convenient nearby sewer point.

There was no mixed refuse in Cape Town, and dry refuse was the only material to which the term " refuse " was applied. Systematic collection

occurred daily except Sundays in the central portion of the city, four times weekly in the more thickly populated suburban areas, three times in the suburban areas, and twice weekly in the outlying districts, where a certain amount of ribbon development had occurred.

Separate waste-paper collection was performed for offices and similar institutions which had accumulations of paper. That paper was sorted, baled, and railed for re-pulping.

TABLE 5.—QUANTITATIVE ANALYSIS OF REFUSE FROM CAPE TOWN

	Av. of 30 towns and cities in England : per cent	Cape Town central area : per cent	Cape Town suburban area : per cent
(*a*) Fine dust (smaller than $\frac{5}{16}$ in.)	36·35	10·7*	23·9*
(*b*) Smaller cinder (between $\frac{5}{16}$ in. and $\frac{3}{4}$ in.)	14·38	22·4†	24·6†
(*c*) Large cinder (larger than $\frac{3}{4}$ in.)	6·25	—	—
(*d*) Vegetable and putrescible matter	13·23	34·1	30·7
(*e*) Waste paper	14·29	21·6	10·1
(*f*) Metals :			
(1) metal containers	3·01	3·2	3·7
(2) other metals	0·99		
(*g*) Rags	1.89	1·5	1·5
(*h*) Glass :			
(1) bottles and jars	2·11	2·4	2·4
(2) broken glass	1·25	2·4	1·9
(*i*) Bone	0·48	1·7	1·2
(*j*) Combustible debris	2·14	(Included under (*a*) and (*b*))	
	100·00	100·0	100·0
Volume : cubic feet per cwt	6·18	Approx. 6–7	Approx. 6–7
Yield per house per week : lb.	37·54	Approximately 54	
Density of refuse :			
cwt per cubic yard	4·37	4	4
lb. per cubic foot	18·15	17	17

* Passing $\frac{1}{4}$ inch. † Passing 1 inch, retained on $\frac{1}{4}$ inch.

Table 5 showed that Cape Town's refuse contained considerably more vegetable matter than that found in the refuse from British cities (more than twice the quantity) and between one-half and three-quarters the quantity of cinders and fine dust. There was more bone in Cape Town refuse, but the quantities of all other ingredients were comparable.

The experience in Cape Town was that flies did not breed in covered refuse-receptacles. The frequency of collection also prevented fly breeding. No separation at the source had as yet been attempted. In England and

Wales, the population was constituted primarily of one race with fairly uniform habits, diet, etc. In Cape Town, there was a heterogeneous population with very different standards of education and living. Separation would involve as a necessity the introduction of bins owned by the local authority.

The yield of refuse per person per day in Cape Town was 1·7 lb., which corresponded exactly with the unit yield for Great Britain.

A weekly collection service did not suffice in a sub-tropical climate such as was experienced in the Cape Peninsula during the summer months, and a minimum of twice weekly was essential, with more frequent collections desirable. That necessity was further accentuated by the high percentage of putrescible matter present in Cape Town refuse.

Trade refuse was removed at a tariff rate, based on actual costs of removal.

Although no enquiry into the organization of the various collection " beats " in Cape Town had been made in recent years, one was about to be made.

Uniforms and protective clothing were supplied. Sample suits of the type specified by the Special Committee of the Institute of Public Cleansing had been ordered to compare them with the present type of protective clothing for desirability and economy.

All mechanical refuse-collection vehicles owned by the City of Cape Town were petrol driven, although a change to diesel units was favoured. The question of using methane gas was being considered, because the City Council had a large supply of gas available from one of its main sewage-disposal works.

All refuse-collection vehicles were rear loading and discharge types, but some had provision for side loading as well and so might be transferred to street-cleansing duties when the occasion demanded. In the suburban areas, animal-drawn vehicles, both side and rear loading, were used as utility wagons for both refuse collections and street cleansing. They had been found both economical and efficient, because in these districts the " haulage distance " from the " beat " to the disposal site was always short.

The question of refuse compression within the vehicle had not been investigated, but Cape Town refuse was known to be compressible to a considerable degree owing to its constitution. At present there was only one area where a unit of that description would be of value, and its introduction was being considered.

The cost of collection in Cape Town was about 24*s* per ton, compared with the Annual Cleansing Cost Return of 8*s* 9*d*. The amount of dry refuse collected per week per man employed was 6 tons. That was lower than the 8 tons average in England, but was brought about partly by the frequent collections made in Cape Town which involved numerous small removals of refuse instead of one large weekly removal.

Methods of tipping adopted in Cape Town conformed to the requirements of all health and cleansing authorities. Refuse was tipped in shallow layers and covered with soil or ash, which formed a suitable seal. Refuse was tipped from above, so that the compressive effect of the collection vehicles was always utilized. In the main tip both a bulldozer and dragline were in use to consolidate and cover the refuse. No mound tipping was carried out. Settlement was about 1 in 4, owing to the compressibility and subsequent decay with reduction in volume of the organic sections of the refuse, and the relatively low proportion of ash and cinders.

The value of controlled tipping in Cape Town had been inestimable. A large area of low-lying marshy ground within 3 miles of the centre of the city had been developed in that way, and was now used as factory sites for industrial development. There were many similar areas in the vicinity of the suburbs of the city which would provide economical and advantageous sites for controlled tipping in the future.

The separation-incineration method of disposal was not yet favoured in Cape Town because it was considered that any heat generated should be utilized in some way. The separation section of the method was being seriously considered, since there was a market for bones, bottles, rags, metals, and paper. The right to salvage those materials had been given to private contractors by public tender. A logical development of that would be the installation of a separation plant, followed by the sale of the salvaged goods.

The direct incineration method of disposal did not, apparently, achieve the production of cheap steam and, since the City of Cape Town already had an Electricity Undertaking, little useful purpose would be served by the adoption of the method.

Pulverization in itself would not be considered as the end point of refuse disposal. It was considered that it should be a preliminary to rapid composting methods.

In England and Wales there was little demand for humus. Some years ago it had been reported that sewage sludge had little manurial value and was not of much consequence as a fertilizer. That was obviously untrue under South African conditions, as was shown by the regular demand for sewage sludge. South African soils were notoriously deficient in humus, and the large farms which were the practice in South Africa usually employed mechanical equipment for many of their agricultural duties. Manures were not easy to obtain and there was a definite need for supplementary fertilizers.

It was stated on p. 117 that: "Given a refuse with a high mineral and a low fermentable content, such as is produced in Britain, a costly rapid fermentation system could hardly be expected to succeed." Since the fermentable content of Cape Town refuse was twice the average figure quoted for Britain and the waste matter much lower in its percentage, it was obvious that after separation a highly fermentable product suitable

for rapid composting would result. The humus-hungry ground would provide a sure market. In Britain all ground was rich in humus, but sometimes lacked certain of the main elements such as nitrogen, phosphorus, and potash, and artificial fertilizers could do much to improve results. Such was not the case in the Cape. Correct conditioning of soil had to be a preliminary to improved cropping, and compost from household refuse offered a source for that conditioning.

Maximum utilization of refuse within sanitary and economic limits would, for Cape Town, mean a composting plant for such of the refuse as could not be economically disposed of by controlled tipping.

The only price Mr Morris could quote for salvaged material was £4 per ton for common mixed paper in bales.

Summarizing his remarks, Mr Morris said that many of the conclusions reached in the Paper were of general interest to countries of temperate climate where a considerable proportion of the refuse was made up of ash and cinders, vegetable residue being in relatively small proportions, and conditions such that weekly collections of refuse sufficed.

In Cape Town the refuse differed materially from that found in England; the population differed in its general character and in the character of the individual. South African soils required compost; soil erosion was an urgent problem. It seemed logical that all economic possibilities to assist in combating soil erosion by the production of compost should be considered.

In conclusion Mr Morris acknowledged the assistance given by Mr T. M. Simpson, M.A., M.Sc., head of the Cleansing and Stables Branch of the City Engineer's Department, for his assistance in the compilation of the information given in the foregoing contribution.

The Author, in reply to Mr Morris's comments on the economics of municipal bin-provision schemes, observed that the important factor was the "average life" of the bin selected. Experience had proved that, given fair and reasonable usage, British Standard bins (B.S. 792) could be expected to last about 8 to 10 years. Those bins were made in three sizes—2, $2\frac{1}{2}$, and $3\frac{1}{4}$ cubic feet capacity—and cost approximately 23, 25, and 29 shillings each respectively. Schemes based on lightly constructed cheap bins could not be expected to succeed.

From his experience of many tests under varying local conditions the Author could not agree that a reliable "yield" or weight figure of refuse collected could be calculated from "volume" data.

The average collection cost of 8*s*. 9*d*. per ton was published in the last Annual Cleansing Cost Return, which was for 1938; the present figure might be as much as 22*s*.

It would be comforting if the humus content of English soils was as satisfactory as indicated, but unfortunately, that was by no means the case in many areas.

From Mr Morris's interesting analysis of Cape Town refuse there

might appear to be a theoretical case for considering a costly rapid-fermentation method of treatment, but the Author would strongly emphasize the importance of securing *a firm and long-term* market for the product at a convenient price *before* a favourable decision was taken.

CORRESPONDENCE
on a Paper published in Proceedings, Part III, December 1952

Paper No. 5887

"Experimental Analysis of Space Structures, with Particular Reference to Braced Domes; with a Note on Stresses in Supporting Ring-Girders" †

by

Zygmunt Stanislaw Makowski, Dip. Ing., and Professor Alfred John Sutton Pippard, M.B.E., D.Sc., M.I.C.E.

Correspondence

Mr T. O. Lazarides observed that the Authors' analysis of the Dome of Discovery was restricted to the case of a single vertical load placed at the summit, and no asymmetrical loading case was mentioned in the Paper. If the aim was to show that experimental analysis carried out on reduced-scale models was a practical alternative to mathematical methods of analysis and could be used for practical design and analysis work, the omission completely invalidated that claim for the following reasons.

As the Authors had pointed out, the Dome of Discovery was a remarkably flat and very highly redundant space frame. Structures of that type were usually very sensitive to the effects of asymmetrically distributed loads acting normally to their surface, for example, asymmetrical snow-load, and that type of loading should, therefore, always be examined with the utmost care by the designer and the analyst, particularly when the edge connexions were of a somewhat unusual type, as was the case with the Dome of Discovery. Owing to the physical properties of structures of that type, the forces and bending moments in various members of the structures could only be obtained analytically—whatever method of analysis was used—in the form of small differences between very large values. The same was presumably true of experimental methods and one would expect that even a very approximate determination of the inner forces caused by an asymmetrical loading would require a degree of experimental precision impossible to attain in practice.

† Proc. Instn Civ. Engrs, Part III, vol. 1, p. 420 (Dec. 1952).

The chief merit of a proposed experimental or other method of structural analysis was that it should yield reliable results of sufficient precision more rapidly than could be obtained by alternative methods. The sensitivity of a structure to certain types of loading was a physical property of the structure and could not be avoided in the analysis. Whenever the analysis of a structure required a large number of elementary relaxation steps—as was the case with the Dome of Discovery—it seemed, therefore, that it would be entirely impossible to analyse it experimentally with the help of a reduced-scale model owing to the unattainable degree of precision required. The Paper did not touch on that point because the only case analysed was a fully-symmetrical-loading case to which the structure was quite insensitive. In the analysis by relaxation, the symmetrical-loading case—own weight—was used as a practice run for testing and refining the methods used for the main task of analysis for an asymmetrical loading ; it was comfortably accomplished by an untrained assistant and certainly would not have justified the trouble and expenditure of model-analysis.

The Authors' omission to consider asymmetrical-loading cases led them to make the entirely misleading statement that for a ring-girder as stiff as the one under consideration it would be legitimate to treat it as an approximately rigid support. That was true for symmetrical loads where the ring-girder worked mainly in tension. For asymmetrical loads, and owing to the special properties of articulated bipod supports, the effective stiffness of the ring-girder was practically nil, because it deformed quite easily in long alternate waves and offered practically no help to the grid. The whole picture was further complicated by the effects of radically dissimilar single and double edge connexions such as points A and points B or F respectively in *Figs 1*. Those effects, examined elsewhere,[6] were vital for the safety of the entire structure and Mr Lazarides did not think that they could have been unravelled by model-analysis; he would appreciate the Authors' opinion on that point.

He pointed out that since the time of the analysis of the Dome of Discovery, new relaxation techniques had been developed by himself, and presumably by others, reducing considerably the tedious work of successive relaxation steps.

The Authors, in reply, stated that they found it difficult to understand the criticism made by Mr Lazarides that the analysis of the Dome of Discovery had been restricted to the case of a single vertical load. It seemed that he had not fully appreciated the method described.

The experimental technique was based on Clerk Maxwell's reciprocal theorem and produced what might be described as an influence " pattern." The actual distribution of loading to which that pattern was applied was immaterial, just as it was when influence lines were used for the solution of beam problems. In the comparative examples given, a symmetrical

[6] See reference 3, p. 440 of the Paper.

case of loading had been taken simply because the arithmetical computation required for comparison with experimental results was simpler than in an asymmetrical case and not because the experimental work had been easier to carry out. Mr Lazarides had, perhaps naturally, concentrated attention on the Dome of Discovery, but for the first model dome (*Figs 3*) the experimental technique had been applied to both symmetrical and asymmetrical loadings; the difference between the experimental and analytical results were of the same order in both cases as shown in Tables 1 (*a*) and (*b*). The only difference in experimental technique was that, to obtain solutions for the asymmetrical loading, the displacements of the node d had been measured, whilst for symmetrical loading they had been measured at node a.

Mr Lazarides obviously distrusted the experimental approach to the solution of structural problems almost as much as most engineers distrusted analytical methods which depended for results on small differences between very large numbers.

The Authors' own belief, after considerable experience of the usefulness of model analysis in practice, and confirmed by the present investigation, was that in many problems it was quite practicable to obtain experimental results of sufficient accuracy for most design purposes both quickly and easily, and the cost should not be high.

It was agreed that the assumption of near-rigidity for the ring girder would probably not be justified in the case of asymmetrical loading, but the assumption was quite unnecessary and had not in fact been used. It had been merely noted as a matter of interest in the particular case. As already indicated, no experimental difficulty arose in asymmetrical loading and when the complete model was tested, as in the case under consideration, the degree of flexibility of the ring girder would look after itself.

In the Dome of Discovery, four sets of influence coefficients (AH, FH, FG, and BG) were sufficient for the estimation of forces in all the bars supported directly on the ring girder, and those could be obtained accurately from the model. The forces upon the ring girder itself could then be readily calculated for any distribution of external loading by using the principle of superposition. If those forces were resolved into tangential, radial, and lateral components, the resultant actions at any section of the ring and the forces in the bipod structure could be calculated by the approximate analytical methods described in the Paper. That method did not require special treatment of the single and double edge connexions at the ring girder as suggested by Mr Lazarides.

Comparison of experimental and arithmetical stress analysis, not only of domes but of many other problems, indicated clearly that the former method had possibilities, only partially explored as yet, that made it a reliable, and to many, an attractive addition to the engineer's equipment. Such methods had been and were being used in many design offices with success. The object of the Authors in presenting the Paper was to show

that the technique was applicable to space as well as plane structures and certainly with no wish to disparage or discourage computational methods. Some designers would always prefer one to the other but the relative merits of the two would have to depend upon the particular problem to be solved.

CORRESPONDENCE

on Works Construction Paper No. 23

"The Application of Precast Concrete to the Construction of Acton Lane 'B' Power Station"†

by

John Anthony Derrington, B.Sc.(Eng.), A.M.I.C.E.,

and

Arthur George Spicer Lance

Correspondence

Mr R. E. G. Gregory expressed the opinion that precast concrete construction should not be applied to power stations.

It was very noticeable that the Authors had referred to advantages in that type of construction without mentioning any disadvantages.

Considerable difficulties would be encountered in arranging for items such as pipe supports, structural alterations for future developments, and last-minute plant requirements. Since the columns were so highly stressed, it would not be considered advisable either to add further direct loads or to cut holes in the concrete columns. The original structural steel scheme had had sufficient design margin to take future additional loads—a point which was found invaluable in practice, and which had saved thousands of pounds on other stations in the past. In many cases, plant contractors' requirements were not known until quite late in the construction programme, and the flexibility of steel design would make it possible often to arrive at an efficient lay-out that could not have been carried out if precast construction had been used.

It should also be remembered that that method of construction could not have been used in the so-called switch annexe if that had been a normal switch annexe. On that site the main switch gear was placed away from the main building and the switch annexe consisted mainly of offices. In

† Proc. Instn Civ. Engrs, Pt III, p. 197 (Aug. 1953).

the normal switch house the floors were so full of holes that tee-beam design would have been out of the question, and the accuracy of construction would not have been sufficient. In spite of the Authors' statement on p. 223 that "tolerances in erection must be kept down below $\frac{1}{8}$ inch" it was found that that degree of tolerance could not be adhered to in practice, as for instance in the case of the precast roof members, where it was found necessary to use a lightweight screed in order to obtain a continuous cross-fall to shed the rain-water; the lintels over the windows where it was found necessary to render the faces to bring them into alignment with the brickwork; and also the main crane beams and columns.

He appreciated that there would be some saving of steel, but the figures mentioned by the Authors did not take into account the fact that the steel design had a reasonable safe design margin for future development and also a higher factor of safety than the precast concrete frame which was stressed to the limit, and in parts even overstressed.

The time-saving factor was again not so clear-cut as the Authors had suggested. Building a power station necessitated a programme in which the delivery of plant and construction of buildings were properly co-ordinated. The only person to benefit from the completion of a building much before its scheduled time was the contractor—he made his profit earlier! The client had to pay the bill and take delivery of an empty building and wait until the plant commenced to arrive (probably late). It was rather unfortunate that at Acton Lane the alternative precast concrete design had held up the placing of the steelwork contract for the boiler house, until the time for steel deliveries had been at its worst. The progress made in the precast concrete scheme had been considerably helped by the fact that the reinforcement had been already available—a point that could not always be guaranteed.

From the foregoing, Mr Gregory drew the conclusion that, whilst precast concrete construction was suitable for simple framed structures such as blocks of flats, etc., the complex nature of power station construction needed the flexibility of design provided by structural steel.

Although he generally preferred reinforced concrete as a medium providing greater aesthetic possibilities, he felt that in the case of a power station a steel structure with, where possible, a precast concrete roof offered advantages that no other type of construction could provide.

The Authors appreciated the points Mr Gregory had raised, which were commonly advanced in favour of structural steel frames, but they believed that both steelwork and concrete (either precast or cast in situ) had their applications, and that they should be considered on their merits in each particular part of the building.

The turbine house of a power station was generally a simple structure in which the architect attempted to achieve clean lines and spaciousness, and to eliminate pipework, ducts, and other unsightly appendages. Structural alterations were unlikely and equally difficult to carry out in

steelwork or reinforced concrete and the Authors thought that Mr Gregory's observations did not apply.

In other parts of the structure, the boiler house, for instance, conditions were different, and although with reasonable foresight preparations might be made for supporting pipes and other services from a concrete frame, it was there that a steel frame would be more readily accepted.

The problem of strengthening or enlarging an existing concrete unit was often dismissed as insoluble when the initial design was considered, although in practice it could be done quite effectively. Repairs and alterations of that nature were becoming more common as opportunities arose. The post-war reconstruction programme included examples of multi-storey buildings with concrete frames, where several damaged columns at ground floor—supporting several floors—had been cut out and replaced in reinforced concrete without any further damage to the building. The Authors were aware that stresses, imposed on the original concrete of a unit by the shrinkage of additional concrete, were less than those placed upon a steel member by the welding-on of additional metal.

The accuracy required in casting, erecting, and plumbing the major structural units was most necessary and was clearly achieved in the structure concerned. It should not be confused with the variations in level of individual roof-units, for few engineers would suggest the omission of a roof screed with any form of construction.

The relative weights of steel for the two schemes had been calculated using the full working stresses in the B.S. Codes of Practice applicable, and undoubtedly, with a lower working stress in the steel, the discrepancy would have been greater.

Mr Gregory's remarks on the higher factor of safety with steelwork construction were difficult to understand, and it would be interesting to see the means by which that statement was substantiated.

The Authors did not appreciate the disadvantages that speedy construction would have, although it was realized that the materials for a reinforced-concrete frame were more easily obtained and undoubtedly helped to speed the job along. They believed that that fact illustrated the " flexibility " of the design chosen, however, and had yet to hear of a similar job in steelwork where the time between the agreement of the design requirements and erection of completed structure had been shorter.

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